ORIGINAL PAPER

A METHOD TO DETERMINE A SINGLE NUMERICAL VALUE FOR REACTIVE POWER IN NON-LINEAR SYSTEMS

ALI KHATI DIZABADI[1], MOHAMMAD J. JAFARIAN[2], AREF DOROUDI[2]

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Abstract. *Classical apparent power decompositions do not fulfill the conditions caused by the presence of harmonics. In harmonic polluted system, the proposed power definitions should be able to determine an accurate value for capacitance power to improve power factor of system. In this paper we modify, by means of instantaneous power equations, the method suggested by Khalsa-Zhang and proposes a new definition for reactive power in non-sinusoidal system. The new definition correctly gives a single numerical equivalent value of reactive power and can be easily applied to calculate the capacitance power. The application of this definition is illustrated with a simple example*

Keywords: *Active power; Non-active power; Reactive power Non-sinusoidal condition.*

1. INTRODUCTION

In 1927, when Budeanu presented his paper as "Reactive and fictitious powers" [1], he has never supposed being the initiator of one of the most controversial issues in power system studies. By widespread use of power electronic loads and harmonic pollution resulted from these devices, his pure theoretical idea has become a practical issue for electrical engineers. Although many theories have been proposed to determine power parameters in non-sinusoidal conditions [2-10], but none of them are widely used in practice and they have not been able to obtain public acceptation. Consequently, the researchers continuously attempt to attain a common definition about this subject. However, a proper and comprehensive theory should have the following characteristics:

1. To present a common definition and formula for power in sinusoidal and non-sinusoidal conditions.
2. Capability of measuring the proposed power decompositions in practical conditions.
3. Can be applied to reactive power compensation and power factor improvement.

Among some of the proposed theories, e.g. Fryze's method [2] does not have the first and third feature, Shepherd and Zakikhani's reactive power definition [4] cannot be applied for power factor calculation and power components nominated by Czarnecki [10] cannot be easily measured and so it doesn't have the second feature. Because of distributed reactive loads in modern power networks and related compensation problems the third feature is so important and if a method does not cover it, the method will not be practical and desirable.

[1] Department of Power,Qaemshahr brances, Islamic Azad University, Qaemshahr, Iran.
E-mail: alikhatidizabadi@yahoo.com
[2] Department of Power Engineering, Faculty of Engineering, Shahed University, Tehran, Iran

Generally engineers in the industry are used to a single numerical equivalent value for the reactive power. But many of the researchers define instantaneous reactive power in the time domain. One of the new methods in this field which was presented by Khalsa – Zhang, based on energy transfer, provides a new formula for reactive power in non-sinusoidal conditions [11]. The method gives a single numerical value for non-active power, but this value is not suitable for reactive power compensator design by simple capacitor. Furthermore the method has some limitations.

In this paper, the Khalsa – Zhang's method is first modified to remove its disadvantages and then based on instantaneous power equations, the method is developed so that one can easily apply it to reactive power compensation. This paper will be continued in the second and third sections by explaining the power formula in non-sinusoidal conditions and reactive power compensation concept. In section four the Khalsa – Zhang's method is introduced and in section five it is developed. Section six presents simulations and finally conclusions are given in section seven.

2. POWER DEFINITION IN UNSINUSOIDAL CONDITIONS

The non-sinusoidal voltage and current generated by nonlinear loads could be analyzed using Fourier series. The non-sinusoidal voltage and current are expressed;

$$V(t)=\sum_{1}^{n}\sqrt{2}V_nSin\left(\omega_n t\right) \quad (1)$$

$$I(t)=\sum_{1}^{n}\sqrt{2}.I_nSin\left(\omega_n t-\alpha_n\right) \quad (2)$$

As in sinusoidal condition the instantaneous apparent power could simply be calculated with product of instantaneous voltages and currents.

$$S(t)=\sum_{1}^{n}\sqrt{2}V_nSin\left(\omega_n t\right)\sum_{1}^{n}\sqrt{2}.I_nSin\left(\omega_n t-\alpha_n\right) \quad (3)$$

Decomposition of instantaneous apparent power in non-sinusoidal conditions is not a clear-cut process, as done for sinusoidal condition [12].

3. REACTIVE POWER COMPENSATION

In pure sinusoidal system, classical apparent power decomposition has been employed to facilitate a tool for system efficiency improvement by means of reactive power compensation. However, the classical decomposition does not fulfill the conditions caused by the presence of non-sinusoidal voltage and/or current. Consequently, new power definitions should be introduced to provide a tool for power factor improvement. In fact, in harmonic polluted system the non-active power is divided into two components; reactive power and distortion power. In this condition, even in the case of pure resistive load, the power factor does not attain unit value. Therefore, in non-sinusoidal conditions compensation reactive power being so highlighted and the proposed power definitions generally bare the aims of accurate and easy determination of maximum power factor realizable with basic capacitive compensation and criterion to measure the degree of load's non-linearity.

4. KHALSA- ZHANG'S METHOD

Khalsa- Zhang's proposed a new definition for active and non-active power that is based on energy transfer and provides a unique solution for them under both sinusoidal and non-sinusoidal conditions. This definition gives a single numerical value of both active and non-active power termed effective active power and effective non-active power. The effective power of instantaneous active or non-active power was defined as the amplitude of an equivalent sinusoidal power waveform that has the same energy transfer as the corresponding active or non-active instantaneous forms in either sinusoidal or non-sinusoidal case. The authors proposed that instantaneous apparent power $(S(t))$ is made up two components: instantaneous active power $(P(t))$ and instantaneous non-active power $(Q(t))$.

$$S(t) = P(t) + Q(t) \tag{4}$$

where $P(t)$ is fully absorbed and consumed by the load, but $Q(t)$ vibrates between source and load. In their theory, the active energy transfer (E_p) due to flow of instantaneous active power (P_{eff}) is defined by:

$$E_p = \int_0^T P(t)dt \tag{5}$$

where T is the fundamental period. The energy transfer due to flow of instantaneous non-active power $(Q(t))$ is bidirectional and cannot be defined in a similar manner to that $P(t)$ using Eq. (5), because its value will be zero. The authors postulated that the positive and negative areas in $Q(t)$ waveform are equal irrespective of the waveform. With this consideration, the non-active energy transfer (E_Q) is defined as twice the area of the positive area:

$$\mathrm{E}_Q = \int_0^{\mathrm{T}} Q_{pos}(t)dt \tag{6}$$

where:

$$\mathrm{Q}_{pos} = \begin{cases} Q(t) & : Q(t) > 0 \\ 0 & : Q(t) < 0 \end{cases} \tag{7}$$

They introduced P_{eff} and Q_{eff} are effective active power and non-active power, respectively as follows:

$$E_p = P_{eff}\, T \tag{8}$$

$$E_Q = \frac{2}{\pi} Q_{eff}\, T \tag{9}$$

Then they concluded:

$$P_{eff} = \frac{E_p}{T} = \frac{1}{T}\int_0^T P(t)dt \tag{10}$$

$$Q_{eff} = \frac{\pi}{2T}E_Q = \frac{\pi}{T}\int_0^T Q_{pos}dt \tag{11}$$

4.1. KHALSA- ZHANG'S METHOD LIMITATIONS

Khalsa- Zhang's definition has a number of limitations as follows:

1. The method requires knowledge of the load impedance and it does not indicate how we can determine the power decompositions when the load is unknown.

2. The definition provides useless information to determine the capacitor power for power factor improvement.

3. The limit of integration in Eq. (5) and Eq. (6) may not be obtained, e.g. when non linear load or system's voltage have a high third harmonic. In this condition and in one period, the instantaneous non-active power cannot be separated into four individual subarea in the time intervals $[0\ T/4]$, $[T/4\ T/2]$, $[T/2\ \ 3T/4]$, and $[3T/4\ T]$. Hence, the energy transfer cannot be obtained by determining the area of $Q(t)$ in the time intervals $[0\ T/4]$ and $[T/2\ 3T/4]$. As an illustration of the last limitation consider the single phase circuit shown in Fig. 1. The circuit consists of a 60 Hz source voltage $(v(t))$ of 110 volts rms and negligible impedance, and a non-linear load which has been shown with a current source (i) and an impedance with $(L = 0.15$ H) and $(R = 0\ \ \Omega)$ (Norton equivalent circuit). In analysis, the current source has only third harmonic with amplitude of 3 A with zero phase angle. Fig. 2 shows the instantaneous non-active power that can be separated in eight unequal regions in one period. As seem, the waveform has four positive portions above the x-axis. Then to determine the energy transfer (by Eq. (6)), the times that $Q(t)$ crosses zero point should be specified. In other words, since the waveform of $Q(t)$ varies with harmonics amplitude and is unknown, the limits of integral of Eq. (6) is not predictable.

5. THE PROPOSED METHOD

In this section, we modified the Khalsa- Zhang's method by presenting a new definition for non-active power. This definition is based on the instantaneous power equations instead of energy transfer. Furthermore, the new definition correctly gives a single numerical equivalent value of reactive power and can be easily applied to compensation problem.

5.1. ACTIVE POWER

We first separate active and non-active powers from each other when type and value of load are unknown. In this case, we propose a current based decomposition in which current is divided into two components namely active current $(I_p(t))$ and non-active current $(I_Q(t))$. Hence:

$$S(t) = V(t)I(t) = V(t)[I_p(t) + I_Q(t)] \quad (12)$$

The instantaneous active power $(P(t))$ is given by:

$$P(t) = V(t).I_p(t) \quad (13)$$

$I_p(t)$ has the same waveform and phase as source voltage and can be calculated as follows [8]:

$$I_p(t) = \frac{V(t)}{R} \times \frac{V_{rms}^2}{V_{rms}^2} \Rightarrow I_p(t) = \frac{V(t)}{V_{rms}^2}.P \quad (14)$$

$$P = \frac{V_{rms}^2}{R} = \frac{1}{T}\int_0^T V(t)I(t)dt \quad (15)$$

where R is load resistance and V_{rms} denotes rms value of voltage. Then by separating $I_p(t)$ from $I(t)$ and substituting it in Eq. (13) one can calculate the active power without knowing about the value of load resistance. On the other hand, any non-sinusoidal instantaneous active power can be equivalent by a sinusoidal active power with amplitude of P_{eff} where P_{eff} is obtained by the following equation:

$$P_{eff} = \sqrt{\frac{1}{T}\int_0^T P^2(t)dt} \quad (16)$$

5.2. NON ACTIVE POWER

Khalsa- Zhang's decomposition inherits a problem which is the lack of direct determination of compensation capacitor power. To achieve this goal, in addition to calculate instantaneous non-active power we should separate the reactive power from instantaneous non-active power. Hence, we first introduce effective apparent power as follows:

$$S_{eff} = \sqrt{\frac{1}{T}\int_0^T S^2(t)dt} \quad (17)$$

Instantaneous non-active power can be derived by subtracting instantaneous active power from instantaneous apparent power:

$$Q(t) = S(t) - P(t) \quad (18)$$

and effective non-active power is given by:

$$Q_{eff} = \sqrt{\frac{1}{T}\int_0^T Q^2(t)dt} \quad (19)$$

With this definition, we don't have to recognize the time interval when $Q(t) > 0$ or the times that $Q(t)$ crosses zero point. We can nominate S_{eff} and Q_{eff} as numerical measures of apparent and non-active power. Instantaneous non-active power is made up two components: instantaneous reactive power that compensate by adding capacitor and instantaneous distortion power which should be constant during the variation of capacitance [8]. Then one can divide the non-active current into:

1. A reactive current component $(I_{Qcom}(t))$ which has the same waveform and phase as integral of source voltage.
2. A distortion current $(I_d(t))$ which remains of the total current after the active and reactive current components have been extracted.

Consequently, if $\tilde{V}$ denotes the rms value of integral of $V(t)$, one can obtain compensated current from integral of active current Eq. (14) and Eq.(15) as follows:

$$I_{Qcom}(t)=\frac{\int V(t)dt}{\tilde{V}^2}\left(\frac{1}{T}\int_0^T\left(\int V(t)dt\right).i(t)dt\right) \tag{20}$$

and the reactive power then is given by:

$$Q_{com}(t)=V(t).I_{Qcom}(t) \tag{21}$$

and effective of this power is:

$$Q_{comeff}=\sqrt{\frac{1}{T}\int_0^T Q_{com}^2 dt} \tag{22}$$

For power factor improvement we can use Q_{comeff} to calculate compensator capacitors power. Instantaneous distortion power and the effective of this power will also be obtained as follows:

$$Q_d(t)=Q(t)-Q_{com}(t) \tag{23}$$

$$Q_{deff}=\sqrt{\frac{1}{T}\int_0^T Q_d^2 dt} \tag{24}$$

This parameter should be constant when we use capacitor for reactive power compensation.

6. COMPUTATION EXAMPLE

Behavior of the proposed power decompositions on reactive power compensation process is analyzed by using the circuit shown in Fig. 1. At first we assumed a sinusoidal voltage source with $f=60$ HZ and $V_{rms}=110\,V$, a linear load with $R=20\Omega$, $L=0.15H$ and $i=0$. The values for S_{eff}, P_{eff}, Q_{eff}, Q_{comeff}, Q_{deff} of the system calculated with the proposed method are given in Table 1.

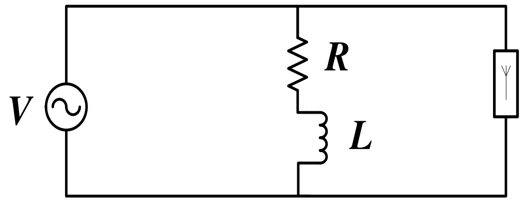

Fig. 1. The circuit used for the analysis.

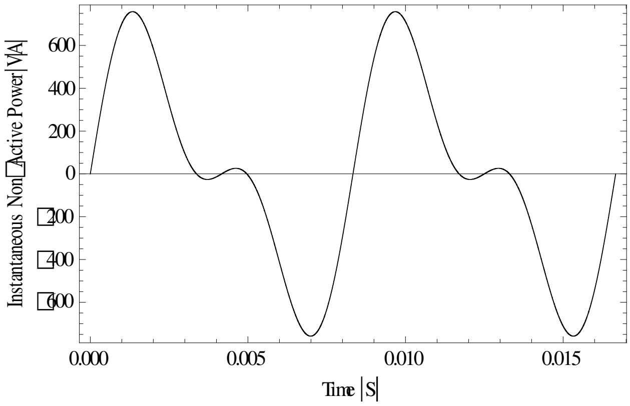

Fig. 2. Instantaneous non-active power of system of Fig. 1.

Table 1. Power decompositions with and without non-linear load.

	S_{eff} (VA)	P_{eff} (Watt)	Q_{eff} (VA)	Q_{comeff} (Var)	Q_{deff} (VA)
Without non-liner load	157.7	82.4	134.5	134.5	0.0
With non-linear load	260.2	82.4	245.0	134.5	145.5

As expected, effective distortion power is zero and effective reactive power is equal to effective non-active power. In second case, we assumed a non-liner load in that current source (i) has 3rd and 5th harmonics with amplitudes of 1A and 0.5A and zero phase angles. Other parameters are as the same as the first case. The results have been also shown in Table 1. As it can be seen, in spite of fixed P_{eff} , the non-active power has changed and is not equal to reactive power.

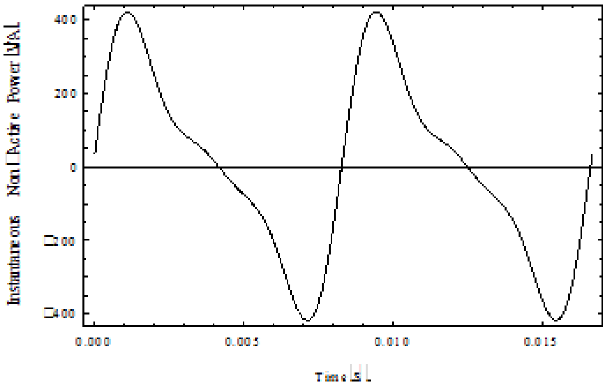

Fig. 3. Instantaneous non-active.

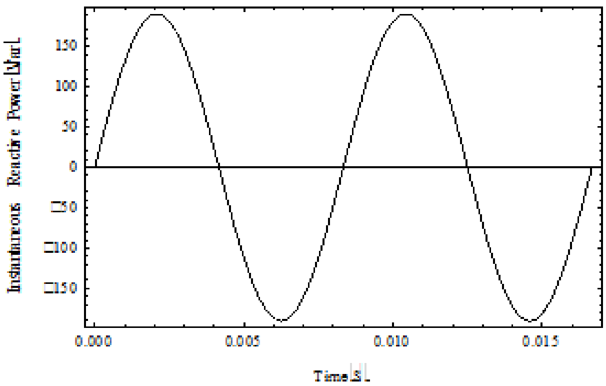

Fig. 4. Instantaneous reactive power.

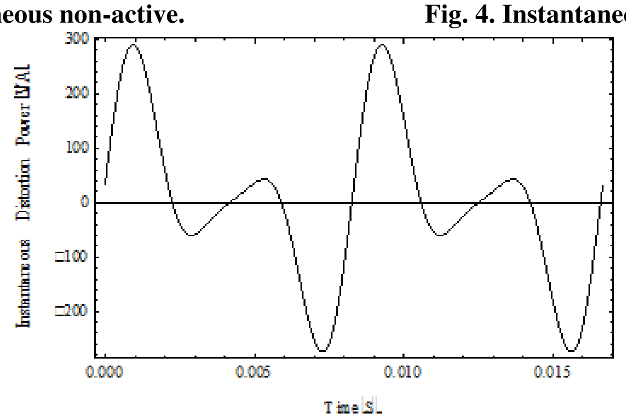

Fig. 5. Instantaneous distortion power.

Figs. 3 – 5 show instantaneous non-active power, instantaneous reactive power and instantaneous distortion power of the system, respectively. The behavior of proposed the power the compositions on reactive power compensation is analyzed with adding a capacitor to the system Fig 6. We assumed two different values for capacitor power. These values are $Q_{cap} = Q_{comeff} = 134.5$ kvar (equals to effective reactive power) and $Q_{cap} = Q_{eff} = 245$ kvar (equals to effective non-active power). The results have been shown in Table 2.

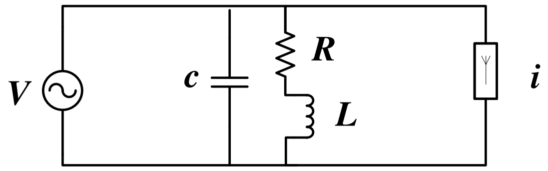

Fig. 6. The circuit used for the analysis.

Table 2. Power decompositions in two deferent values for capacitor power.

	I_c (A)	S_{eff} (VA)	P (Watt)	Q_{eff} (VA)	Q_{comeff} (Var)	Q_{deff} (VA)
Q_{comeff} =134.5 (kvar)	1.73	169.7	82.4	145.5	0	145.5
Q_{comeff} =245 (kvar)	2.06	157.6	82.4	131.1	32.7	145.5

Note that for the case $Q_{cap} = 134.5$ kvar, the capacitor current can be obtained from Eq. (20). On the other hand, in the case of $Q_{cap} = 234.5$ kvar, the capacitive current should be obtained from the follwing equation :

$$I_c = \sqrt{\frac{1}{T}\int_0^T \left(I(t) - I_p(t)\right)^2} \tag{25}$$

The results comparison of the two cases show that with our proposed method the reactive power completely compensated whereas we cannot achieve to this goal by effective non-active power and since in second case the capacitance power has been chosen higher than that we need, source current leads source voltage and therefore the system goes to capacitive mode. Fig. 7 shows the instantaneous fundamental current and voltage of the source without compensator capacitor. As it can be seen, the current lags source voltage. Figs. 8 and 9 show the instantaneous fundamental currents and voltages with compensator capacitor included in two cases $Q_c = 134.5$ kvar, $Q_c = 245$ kvar.

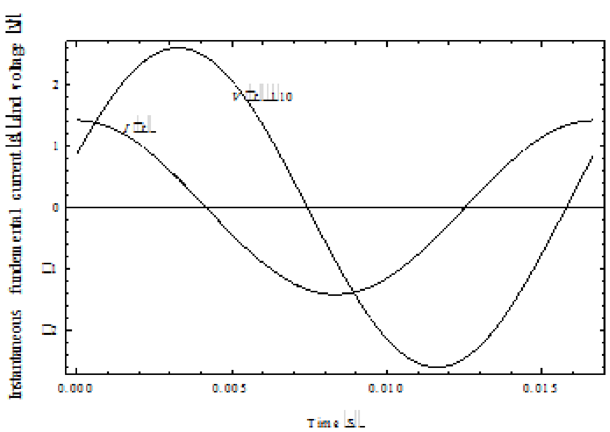

Fig. 7. Instantaneous fundamental current without compensator capacitor.

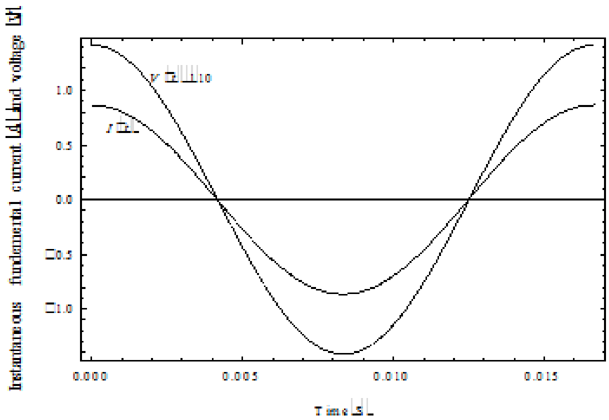

Fig. 8. Instantaneous fundamental current with compensator capacitor in first case.

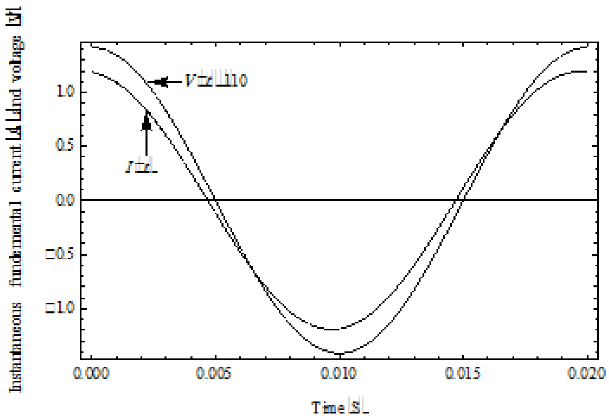

Fig. 9. Instantaneous fundamental current with compensator capacitor in second case.

Fig. 8 reveals that by correct compensating the instantaneous fundamental current and voltage cross the zero point simultaneously, but as shown in Fig. 9 incorrect compensation causes the current leads the voltage. On the other hand, table I shows that the distortion power has not changed in both cases after adding the capacitor to the system.

6. CONCLUSION

In this paper we develop the KHalsa-Zhang method to propose a new definition for reactive power in non-sinusoidal conditions. The new definition can be directly applied to calculate the capacitance power for reactive power compensation and power factor improvement. A simple phase circuit was used and the results showed the new definition correctly provides a single value for capacitor power.

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