

# Position dependent circuit model for thin avalanche photodiodes

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**Abstract** This paper presents a circuit model for thin avalanche photodiodes (APDs). In this model, the nonuniformity of the electric field in the multiplication region is modeled using a stepwise method. The model also tries to take the effects of carrier's position dependent properties, like carrier's dead length and the history of carrier's previous ionization into account by developing an effective electric field in the multiplication region. The output photocurrent and multiplication gain obtained from the proposed model for different lengths of the multiplication region achieve a good agreement in comparison with available experimental data. In addition, calculated excess noise factor reveals the model ability for noise and sensitivity analysis.

**Keywords** avalanche photodiode (APD), circuit modeling, multiplication gain, nonuniform electric field, excess noise factor

## 1 Introduction

Avalanche photodiodes (APDs) have wide application in optical communication systems due to their built-in gain, and it can yield high sensitivity for the receive side [1]. Many efforts have been made for circuit model of APDs, since such circuit model can simplify design of a whole transceiver in a single environment [2]. Either rate-equations [3], or current continuity equations [4], can be considered as the basis for APD's circuit model. However, APD's circuit model by these two equations usually ignore position dependent factors, such as nonuniformity of electric field, carrier dead length and previous ionization history, and this makes inaccurate for modeling thin APDs, which attract many attentions due to their superior noise performance. There were also some efforts to take the

effect of dead length into account [5] and nonuniform electric profile as well as carrier dead length [6]. However, both dismiss field-dependent carrier's previous ionization history. In this paper, we introduced a circuit model based on position dependent multiplication gain calculation, in which carrier's dead length and the history of carrier's previous ionization were included by an effective electric field for electrons and holes.

## 2 Theory

A schematic diagram of a typical p-i-n APD (PIN-APD) photodetector is shown in Fig. 1. Light is incident on the n-side of the junction. A nonuniform electric field profile resulting from a reverse bias voltage and its stepwise approximation are also illustrated in Fig. 1. According to one-dimensional Poisson's equation and using depletion approximation, the electric field profile in the lightly doped i-region can be written as

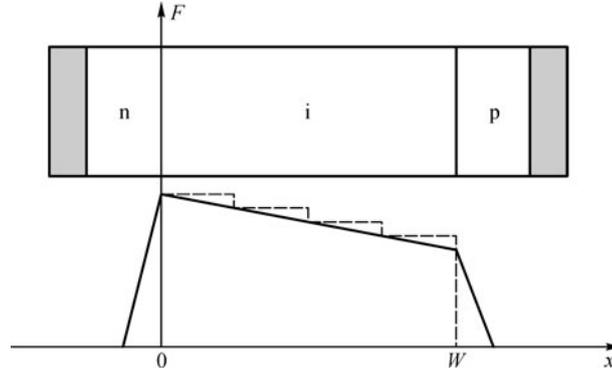
$$F(x) = qN_p x_p / \epsilon_0 \epsilon_s - qN_i x / \epsilon_0 \epsilon_s, \quad (1)$$

where  $q$  is the electron charge,  $\epsilon_0$  and  $\epsilon_s$  are the vacuum permittivity and the semiconductor relative permittivity respectively,  $N_p$  and  $N_i$  are the doping concentration in the p- and i-region respectively, and  $x_p$  is the depletion region width in the p-region obtained as given in Ref. [6].

The position-dependent gain defined as the ratio of the number of final electron-hole pairs to that created initially in the i-region, is given by [7]

$$M = \frac{\exp\left(-\int_0^W (\alpha(x) - \beta(x)) dx\right)}{1 - \int_0^W \alpha(x) \exp\left(-\int_x^W (\alpha(x') - \beta(x')) dx'\right) dx}, \quad (2)$$

where  $W$  represents the i-region thickness,  $\alpha(x)$  and  $\beta(x)$  are the electron and hole ionization coefficients respectively [8]:



**Fig. 1** Schematic diagram with electric field profile of a p-i-n avalanche photodiode (PIN-APD)

$$\alpha(x) = a_n \exp(-b_n/F)^{c_n}, \quad (3)$$

$$\beta(x) = a_p \exp(-b_p/F)^{c_p}, \quad (4)$$

where  $F$  ( $\text{V} \cdot \text{cm}^{-1}$ ) is the position-dependent electric field,  $a_n$  ( $\text{cm}^{-1}$ ) and  $a_p$  ( $\text{cm}^{-1}$ ),  $b_n$  ( $\text{V} \cdot \text{cm}^{-1}$ ) and  $b_p$  ( $\text{V} \cdot \text{cm}^{-1}$ ),  $c_n$  and  $c_p$  are all material constants.

To begin the circuit modeling procedure, we define  $N(x)$  as a position dependent integration term and then approximate it using trapezoidal rule:

$$\begin{aligned} N(x) &= \int_x^W [\alpha(x') - \beta(x')] dx' \\ &\approx \frac{W-x}{2m} \left\{ [\alpha(x) - \beta(x)] \right. \\ &\quad \left. + 2 \sum_{k=1}^{m-1} \left( \alpha\left(k \frac{W-x}{m}\right) - \beta\left(k \frac{W-x}{m}\right) \right) \right. \\ &\quad \left. + [\alpha(W) - \beta(W)] \right\}, \end{aligned} \quad (5)$$

where, it is assumed that the i-region from  $x$  to  $W$  is subdivided into  $m$  equal segments. As shown in Fig. 2(a), each  $\alpha - \beta$  term in Eq. (5) can be modeled with a current source [e.g.,  $I(x) = \alpha(x) - \beta(x)$ ], all passing through a resistor  $R_N$  with a value of  $(W-x)/2m$ , where  $N(x)$  appears as a node voltage. Observing Eq. (2), for calculating  $M$ , we need the exponential of  $N(x)$ , where  $D_N$  in Fig. 2(a) carries out the exponential function as below:

$$I_N(x) = I_0 \exp[V_N(x)/nV_T] = \exp[-N(x)], \quad (6)$$

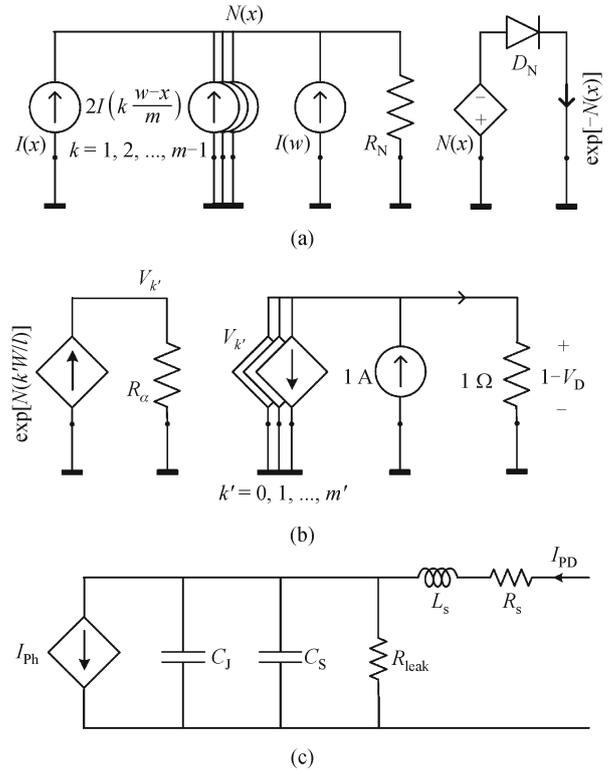
where  $I_0$  is given a value of 1 A and the effect of  $nV_T$  is removed by modifying the value of  $R_N$  to

$$R_N = nV_T(W-x)/2m.$$

Based on the explained method, the integration term in the denominator of Eq. (2) can be approximated as

$$V_D = \int_0^W \alpha(x) \exp\left(\int_x^W (\alpha(x') - \beta(x')) dx'\right) dx$$

$$\begin{aligned} &= \int_0^W \alpha(x) \exp[N(x)] dx \\ &\approx \frac{W}{2m'} \left\{ \alpha(0) \exp[N(0)] \right. \\ &\quad \left. + 2 \sum_{k'=1}^{m'-1} \alpha(k' W/m') \exp[N(k' W/m')] \right. \\ &\quad \left. + \alpha(W) \exp[N(W)] \right\}, \end{aligned} \quad (7)$$



**Fig. 2** Equivalent circuit model for avalanche photodiodes based on multiplication gain calculation. (a) Calculating  $N(x)$  and  $\exp[-N(x)]$ ; (b) calculating  $V_D$ ; (c) output terminal of the device

where, it is assumed that the i-region thickness is subdivided to  $m'$  equal segments where each segment again divided to  $m$  segment (required to calculate  $N(x)$  in each segment). As shown in Fig. 2(b), each  $\exp[N(x)]$  term can be realized as a current using a circuit like that given in Fig. 2, requiring  $m' + 1$  circuit totally. Meanwhile, a multiplication factor of  $\alpha(x)$  is needed for each term, which defining  $x = k'W/m'$  can be realized by  $R_\alpha = W\alpha(x)/2m'$  for  $k' = 0$  and  $m'$ , and twice of that for  $k' = 1$  to  $m' - 1$ .

Now, according to Eq. (2), we have

$$M = \exp[-N(0)]/(1 - V_D).$$

Once  $M$  is known, the multiplied photocurrent response can be obtained as (Ref. [9])

$$I_{Ph} = q\eta \frac{P_{opt}}{h\nu} M, \tag{8}$$

where  $P_{opt}$  is the incident optical power and  $\eta$  is the quantum efficiency defined as (Ref. [9])

$$\eta = \eta_i(1 - R)e^{-\alpha_p d} \left( 1 - \frac{e^{-\alpha_i W}}{1 + \alpha_i L_p} \right), \tag{9}$$

where  $\eta_i$  is the internal quantum efficiency,  $R$  is the facet reflectivity of the p-region,  $\alpha_p$  and  $\alpha_i$  are the absorption coefficient of the p- and i-region, respectively,  $d$  is the width of p-region and  $L_p$  is the diffusion length for holes. Figure 2(c) resembles the output terminal of the device. In this figure,

$$I_{PD} = I_{Ph} + (C_J + C_S) \frac{dV_A}{dt} + \frac{V_A}{R_{leak}}, \tag{10}$$

where  $I_{PD}$  is the total output current of the device,  $V_A$  is the bias voltage over the junction,  $R_{leak}$  is the parallel parasitic resistance of the APD, and  $C_S$  and  $C_J = \epsilon_0 \epsilon_s A / (x_n + x_p + W)$  are the parasitic and junction capacitances.

### 3 Results and discussion

To reach to a more accurate circuit model for thin APDs, we should consider the carrier's dead length as well as the history of carrier's previous ionization, both depend on the electric field. These effects can be taken into account by defining an effective electric field for electrons and holes [7,10] as

$$F_{eff,e}(x) = \int_x^{x'=x-l_e} dx'' F(x'') f_{ce}(x''), \tag{11}$$

$$F_{eff,h}(x) = \int_x^{x'=x+l_h} dx'' F(x'') f_{ch}(x''), \tag{12}$$

where  $l_e$  and  $l_h$  are electron's and hole's dead length respectively, and  $f_{ce}(x)$  and  $f_{ch}(x)$  are correlation functions defined as (Refs. [7,10])

$$f_{ce,h}(x) = \frac{2}{\sqrt{\pi}\lambda_{e,h}} \exp\left(-\frac{(x' - x)^2}{\lambda_{e,h}^2}\right) \approx \frac{2}{\sqrt{\pi}\lambda_{e,h}} \exp\left(-\frac{l_{e,h}^2}{\lambda_{e,h}^2}\right). \tag{13}$$

In these equations,  $\lambda_e$  and  $\lambda_h$  are correlation length for electrons and holes respectively obtained as (Ref. [7])

$$\lambda_{e,h} \approx l_{e,h} = V_{de,h}/F(x), \tag{14}$$

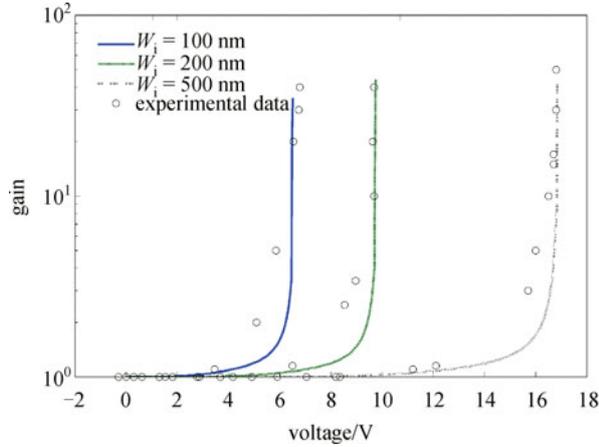
where  $V_{de}$  and  $V_{dh}$  are voltage drops across the electron's and hole's dead length, respectively.

To verify the model capability for predicting  $M$ , a homojunction GaAs PIN-APD has been simulated for different i-region thickness and the results have been compared with experimental data presented in Ref. [7]. In this model, universal width-independent parameters given in Ref. [8] are utilized for ionization coefficients. The parameters used in simulation are tabulated in Table 1. The results are shown in Fig. 3. This comparison shows that our model resembles the experimental data in an acceptable manner especially the value of breakdown voltage. However, near breakdown the model underestimates the gain. This is mainly due to ignorance of field in cladding layers, and possibly the problem of inaccurate estimation of field in GaAs devices.

To justify the model accuracy more, as a second example, an InAlAs-APD with an area of about  $31415 \mu\text{m}^2$  is modeled and simulated. As shown in Fig. 4, the results for photocurrent are in excellent agreement with the

**Table 1** Parameters used in simulations

symbol	value		unit
	GaAs	InAlAs	
$A$	31415	31415	$\mu\text{m}^2$
$a_n$	$6.01 \times 10^6$	$4.17 \times 10^6$	$\text{cm}^{-1}$
$b_n$	$2.39 \times 10^6$	$2.09 \times 10^6$	$\text{V} \cdot \text{cm}^{-1}$
$c_n$	$0.9 \times 10^6$	$1.2 \times 10^6$	
$\alpha_p$	$3.59 \times 10^6$	$2.65 \times 10^6$	$\text{cm}^{-1}$
$b_p$	$2.26 \times 10^6$	$2.79 \times 10^6$	$\text{V} \cdot \text{cm}^{-1}$
$c_p$	$0.92 \times 10^6$	$1.07 \times 10^6$	
$N_i$	$1 \times 10^{16}$	$1 \times 10^{16}$	$\text{cm}^{-3}$
$N_n$	$5 \times 10^{18}$	$2 \times 10^{18}$	$\text{cm}^{-3}$
$N_p$	$5 \times 10^{18}$	$2 \times 10^{18}$	$\text{cm}^{-3}$
$W_i$	100–800	100–799	nm
$W_n$	1000	200	nm
$W_p$	1000	300	nm
$R_s$	5	20	$\Omega$
$L_s$	0.3	0.3	nH
$R_{leak}$	$10^{10}$	$10^{10}$	$\Omega$
$C_s$	0.2	0.25	pF

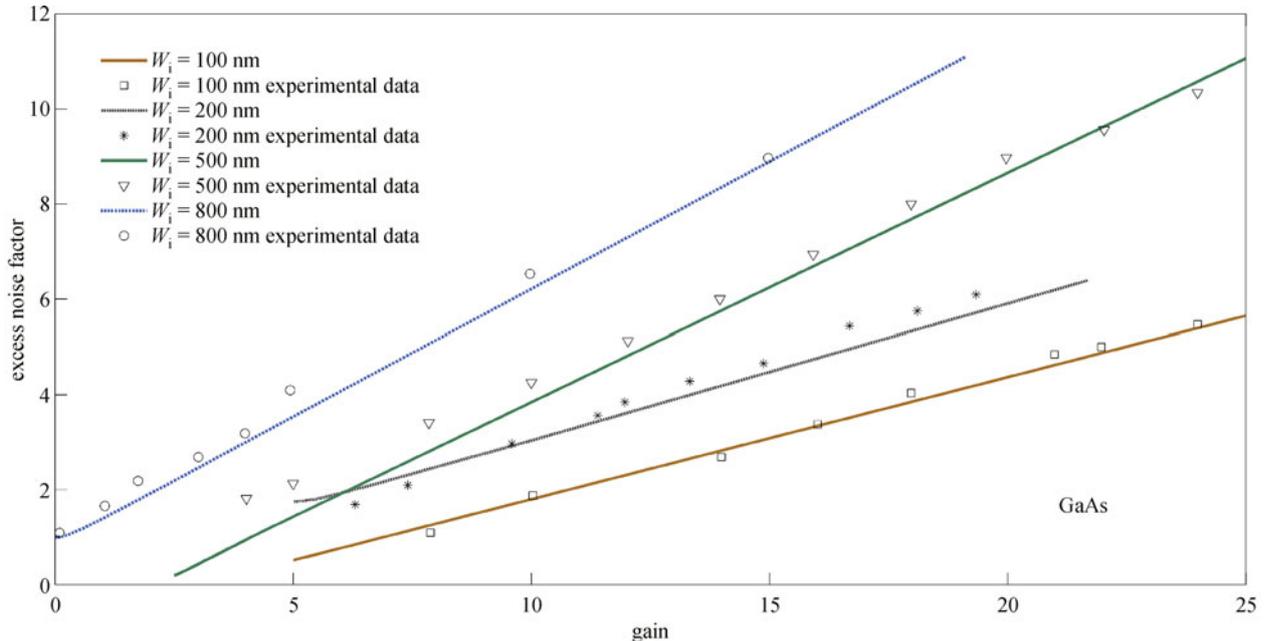


**Fig. 3** Multiplication gain as function of reverse bias voltage for homojunction GaAs APDs with i-region of  $W_i = 100, 200$  and  $500$  nm (solid-, dashed-, and dotted-lines) compared to experimental data reported in Ref. [7]

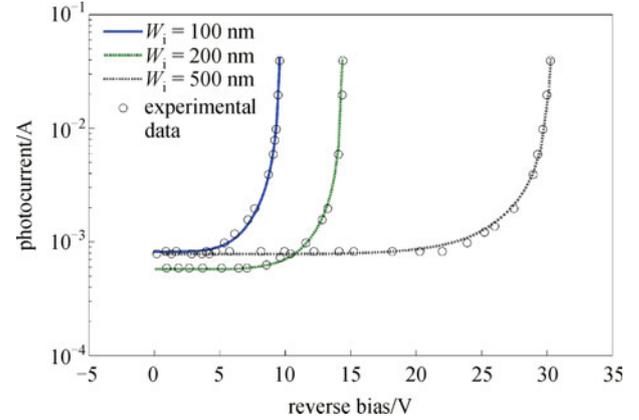
experimental data reported in Ref. [11], as the model provides almost accurate estimation for current levels and breakdown voltages. This result indicates that accounting for carrier's dead length and previous ionization history is a key factor in circuit modeling of thin APDs.

Figure 5 represents excess noise factor versus gain for a GaAs APDs with different thickness of i-region based on the following relation assuming only electron injection at  $x = W$  [7],

$$F(M) = kM + (1-k) \left( 2 - \frac{1}{M} \right). \quad (15)$$

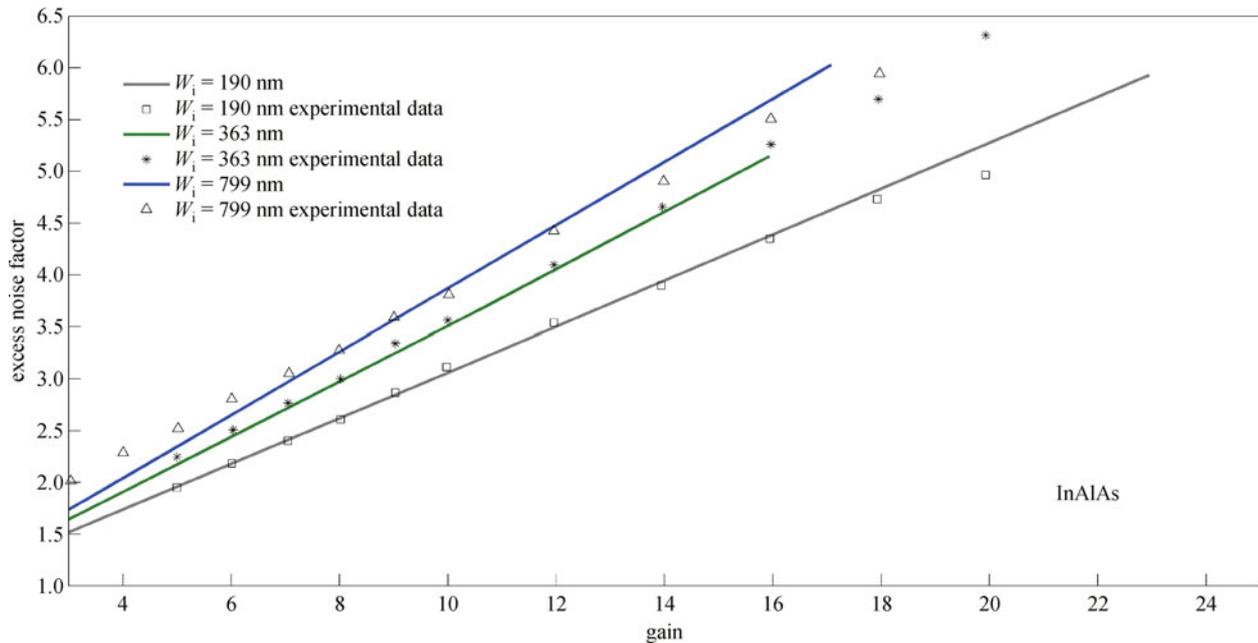


**Fig. 5** Comparison of simulation and experimental results of excess noise factor of a GaAs APDs [6] for different thickness of i-region;  $W_i = 100, 200, 500$  and  $800$  nm



**Fig. 4** Photocurrent as function of reverse bias voltage for homojunction InAlAs APDs with i-region of  $W_i = 100, 200$  and  $500$  nm (solid-, dashed-, and dotted-lines) compared to experimental data presented in Ref. [11]

Simulation results carried out for  $W_i = 100, 200, 500$  and  $800$  nm reveal an acceptable agreement with experimental data afforded by Ref. [8]. In Fig. 5, however, for most of widths of i-region, the noise has been lightly underestimated that can be referred to underestimation of gain in these GaAs devices. To certify the model ability for estimation of excess noise factor, it is tested with another InAlAs APD, for  $W_i = 190, 363$  and  $799$  nm. As exhibited in Fig. 6, results of simulated excess noise factor are close to the empirical data [8] for low gain and high gain situations. Without developing an effective electric field in the multiplication region, the predicted excess noise factor will be overestimated. In fact, considering the dead length,



**Fig. 6** Comparison of simulation and experimental results of excess noise factor of an InAlAs APDs [8] for different thickness of  $i$ -region;  $W_i = 190, 363$  and  $799$  nm

in which carriers gain sufficient energy to have a non-negligible ionization probability helps the model to predict the multiplication gain and resulting excess noise factor accurately.

## 4 Conclusions

This paper has proposed a new and simple circuit model for thin APDs suitable for performance evaluation of optical systems. The model can be used for excess noise factor calculation. The main contribution of this work is in consideration of nonuniform electric field profile, carrier's dead length and previous ionization history, simultaneously. The model can be simply extended to other types of photodetectors like separate absorption, grading, charge, and multiplication APD (SAGCM-APD). Simulation indicates that accounting for nonlocal effects using a modified electric field results in a better estimation of the APD characteristics as the model provides excellent agreement with published experimental data.

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