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ORIGINAL ARTICLE

A new link function in GLM-based control charts to improve monitoring of two-stage processes with Poisson response

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Abstract In this paper, a new procedure is developed to monitor a two-stage process with a second stage Poisson quality characteristic. In the proposed method, log and square root link functions are first combined to introduce a new link function that establishes a relationship between the Poisson variable of the second stage and the quality characteristic of the first stage. Then, the standardized residual statistic, which is independent of the quality characteristic in the previous stage and follows approximately standardized normal distribution, is computed based on the proposed link function. Then, Shewhart and exponentially weighted moving average (EWMA) cause-selecting charts are utilized to monitor standardized residuals. Finally, two examples and a case study with a Poisson response variable are investigated, and the performance of the charts is evaluated by using average run length (*ARL*) criterion in comparison with the best literature method.

Keywords Two-stage processes · Generalized linear models (GLM) · Poisson response variable · Cause-selecting control chart (CSC) · Log square root link function · Average run length (*ARL*) · Standardized residual (*SR*)

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1 Introduction

Many industrial processes include different stages, where subproducts are produced in stages to produce final products. In these processes, stages are dependent on each other due to the cascade property, where changes in quality characteristics of previous stages affect the quality characteristic of the current stage. These processes are so-called multistage processes. There are many approaches to monitor multistage processes. One approach is to use the standard Shewhart control chart to monitor each stage separately. This approach is not appropriate because an out-of-control signal of the current stage could be due to the effects of previous stages as well as the current stage. The other approach is using a multivariate or multiattribute control chart such as Hotelling's T^2 to monitor the quality characteristics of all steps simultaneously. This approach is not appropriate either, because when an out-of-control signal is observed, it is difficult to determine the stage responsible for the signal [1].

Mandel [2] proposed the regression-adjusted control chart to monitor multistage processes. Zhang [3] proposed a cause-selecting control chart (CSC) suitable for multistage process monitoring. Lucas and Saccucci [4] proposed the use of an exponentially weighted moving average (EWMA) control chart, namely EWMA CSC, to monitor the residuals of the normal variable in the second stage. Hawkins [5, 6] extended Mandel's method [2] and proposed control charts based on regression adjustment variables. Wade and Woodall [1] presented a new CSC with prediction limit and showed that their method improves the statistical performance of the control chart. Hauck et al. [7] extended the model of Hawkins [6] to monitor two-stage processes with multicorrelated variables in each stage. Amiri et al. [8] considered a two-stage process with a normal variable in each stage. They used the MLE approach to estimate step change point in the mean of multistage process in phase II and showed suitable performance of the proposed estimator through simulation studies.

In some situations, there are outliers in historical data used for estimating the relation between the quality characteristics in two-stage processes. Outliers decrease the performance of monitoring procedures and CSC control charts because the regression model that relates the stages is distorted, and consequently, the control limits are stretched. Recently, Asadzadeh et al. [9, 10] considered outliers in historical data and showed their effect on the least squares method when the output variables follow normal distribution in a two-stage process. They used robust monitoring approaches to establish the relation between quality characteristics in different stages. Finally, they compared the robust and non-robust schemes in terms of the average run length criterion. They also showed the power of their robust scheme in this condition. Moreover, Asadzadeh et al. [11] assumed a two-stage process with outliers in historical data in the quality characteristic of each stage. They considered a robust fitting procedure based on a compound estimator to establish the relationship between the quality characteristics. They presented a robust monitoring approach, and showed suitable performance of their proposed robust scheme in terms of the average run length criterion through simulation studies.

Although the design parameters of control chart are usually assumed fixed in traditional SPC techniques, but there are some schemes known as adaptive control chart in which these parameters (namely sample size, sampling interval, and control limits) are varying. In some two-stage process monitoring works, researchers assumed design parameters of the cause-selecting control chart are not fixed. To name a few works, Yang and Su [12] assumed the sample size in a two-stage process is variable and proposed variable sample size (VSS) CSC to monitor the process. They used the adjusted average time to signal (AATS) to investigate the performance of the proposed VSS CSC. Yang and Su [13] considered the variable sampling interval (VSI) control chart for a two-stage process. They showed the performance of the proposed VSI CSC in terms of the AATS criterion. Moreover, Yang and Su [14] considered both VSS and VSI properties and designed the variable sample size and sampling interval (VSSI) CSC for monitoring of a two-stage process. They also used the AATS criterion to evaluate the performance of the proposed control chart.

The observations collected from a process output are not always independent, and in many processes such as chemical processes, observations are auto-correlated. Besides, there are measurement errors due to the measurement systems. In these situations, the performance of control charts being used to monitor the process can get unfavorable effects. Yang and Yang [15, 16] considered a two-stage process with a single quality characteristic in each stage. The aim of their research was to monitor the mean of the quality characteristic in the first as well as the second stages. They assumed that the auto-correlated observation in the first stage can be modeled by ARMA(1,1) and AR(1) models. Then, they proposed the CSC control chart for residuals to monitor a two-stage process with auto-correlated data. They compared the performance of their proposed control charts in terms of the rate of true or false alarms to the ones of the Hotelling T^2 control chart as well as individual Shewhart charts.

In most research works in the field of multistage processes, authors assume a two-stage process with normal quality characteristics in each stage and use the aforementioned methods to monitor quality characteristics. However, real-world industrial processes may have quality characteristics following non-normal distributions. In this regard, Jearkpaporn et al. [17] assumed the response variable or quality characteristic of the second stage to follow a gamma distribution, considered the log link function to relate the response and input variables, and obtained deviance residuals by generalized likelihood ratio (GLR) statistic for response variables. Then, a generalized linear model (GLM)-based control chart was used to monitor the deviance residuals. Skinner et al. [18] assumed response variables to follow a Poisson distribution and used a log link function to obtain deviance residuals by GLR statistic. They showed that the distribution of deviance residuals is approximately normal and then designed a control chart to monitor the statistic. Skinner et al. [19] used a GLM-based control chart to monitor a semiconductor process with multiple inputs and outputs with varying relationships. In this process, the output variables are assumed to follow Poisson distributions. They showed that the GLM-based chart has better performance than multiple C charts in detecting changes in the means of output variables. Jearkpaporn et al. [20] considered a multiple-stage process with three variables in each stage. They assumed that variables in each stage follow gamma, normal, and Poisson distributions. They used a GLM-based method to relate variables and showed that GLM-based control charts are suitable to detect changes in the mean values of quality characteristics in the second and third stages. Amiri et al. [21] developed a cause-selecting control chart based on the standardized residuals of a generalized linear model to monitor a two-stage process with a Poisson-distributed quality characteristic in the second stage. They investigated the performance of their proposed control chart in terms of average run length criterion under two different link functions in comparison with the method by Skinner et al. [18]. The results showed the better performance of their proposed control chart in detecting increasing shifts. Amiri et al. [22] considered a two-stage process with a gamma quality characteristic in the second stage and used a cause-selecting control chart to monitor the residuals of this quality characteristic obtained by GLMs and the normal-to-anything (NORTA) inverse method. The performance of their proposed control chart was investigated in terms of average run length (*ARL*) criterion in comparison with the one obtained by the method of Jearkpaporn et al. [17]. The results illustrated the better performance of their method in detecting increasing and decreasing shifts.

Aghaie et al. [23] first considered a two-stage process with a Bernoulli variable at the first stage and a Poisson variable at the second stage. Then, they suggested a new method based on the generalized Poisson distribution to monitor the process. Yang and Yeh [24] proposed a cause-selecting chart to monitor two-stage processes with a binary variable in each stage. They first assumed that the paired data could only be obtained at the end of the process and that these paired data follow a bivariate binary distribution. Then, they related the outgoing quality characteristic on incoming variables by the arcsin-transformed model. Finally, they monitored the obtained residuals using a CSC. Shang et al. [25] considered a multistage process with binomial (binary) data and modeled it with a binary state space model (BSSM). They extended corresponding monitoring and diagnosis schemes by applying a hierarchical likelihood approach and directional information from BSSM. They showed the better performance of the proposed schemes by simulation in comparison with the χ^2 scheme.

Some researchers investigated monitoring product reliability in multistage processes with cascade property [26–29]. In these papers, the output variable follows the Weibull distribution. Asadzadeh et al. [30] proposed a method to monitor a multistage process with autocorrelated reliability data. They modified the proportional hazards (PH) models to justify the effect of cascade property in line with the autocorrelation.

In some processes, quality is described by a relationship between quality characteristics known as a profile. In this regard, Ghahyazi et al. [31] considered a simple linear profile in a multistage process with cascade property in phase II. They evaluated the cascade effect on the performance of the T^2 control chart and showed that the performance of the T^2 control chart deteriorates. Then, they eliminated the cascade property by using the U statistic and presented a modified control scheme for monitoring the simple linear profile in two-stage processes.

In this paper, a two-stage process with a Poisson quality characteristic in the second stage is considered. A new link function is proposed to first establish the relationship between the response and input variables using a regression function. We assume that the parameters of the regression function are known, and hence, we are in phase II analysis. Then, a standardized residual (SR) statistic that follows a standard normal distribution is proposed based on the proposed link function. Next, two CSC control charts are developed to monitor the proposed SR statistic. Finally, the performance of the proposed method is investigated through two examples in terms of ARL criterion in comparison with the r-control charts introduced by Skinner et al. [18].

This paper is organized as follows: In Section 2, a brief background on the generalized linear models and popular link functions for Poisson responses is first presented. Then, a new link function and SR statistic for a Poisson response variable are proposed. In Section 3, the control chart to monitor the proposed SR statistic is developed. In Section 4, two examples are considered to investigate the performance of the proposed method in comparison with the method of Skinner et al. [18]. In addition, in Section 5, a case study from Skinner et al. [18] is considered to illustrate the performance of the proposed methods. Our concluding remarks are given in Section 6.

2 A brief background

The performance of the ordinary least squares (OLS) method deteriorates when the response variable of a process is non-normal. The GLM is an appropriate method to model response variables with the exponential family. The exponential family includes Poisson, binomial, normal, gamma, exponential, and inverse normal distributions. In this case, the mean of the response variable, μ, links to a linear combination of the input variables, $\mathbf{x}'\boldsymbol{\beta}$, by a link function as

$$w(\mu) = \mathbf{x}'\boldsymbol{\beta}, \tag{1}$$

where $\boldsymbol{x}$ is a vector of the input variable and $\boldsymbol{\beta}$ is a vector of regression coefficients. The mean of response variable is given by the inverse of the link function, $w(\mu)$. For example, if one uses a log link function for the mean of a Poisson response λ as $\log(\lambda)=\mathbf{x}'\boldsymbol{\beta}$, then λ is obtained by Eq. 1.

$$\lambda = e^{\mathbf{x}'\boldsymbol{\beta}}. \tag{2}$$

Skinner et al. [18] used a log link function and obtained their GLM-based statistic as

$$r_{\mathrm{j}} = \operatorname{sign}\left[y_{\mathrm{j}}-\exp\left(\mathbf{x}'\boldsymbol{\beta}\right)\right]\left\{2\left[y_{\mathrm{j}}\ln\left(\frac{y_{\mathrm{j}}}{\exp(\mathbf{x}'\boldsymbol{\beta})}\right)-\left[y_{\mathrm{j}}-\exp\left(\mathbf{x}'\boldsymbol{\beta}\right)\right]\right]\right\}^{\frac{1}{2}}. \tag{3}$$

Moreover, considering an additive shifts, they obtained the upper control limit to monitor the deviance residuals as

$$UCL = \bar{r} + k\sqrt{s^2(r)}, \tag{4}$$

where $\bar{r}$ and $s^2(r)$ are the sample mean and variance of the residuals, respectively, and k is the coefficient of control limits determined such that we achieve a predetermined in-control ARL.

The other link function used for the Poisson response variable is the square root link ($\sqrt{\lambda} = \mathbf{x}'\boldsymbol{\beta}$). In this case, the mean of the Poisson variable λ is obtained by

$$\lambda = (\mathbf{x}'\boldsymbol{\beta})^2. \tag{5}$$

3 Proposed method

In this section, a new link function for a Poisson response is first introduced by a combination of two existing link functions. Then, the standardized residuals are obtained, where their distribution is investigated. Finally, the proposed control charts are explained.

3.1 The proposed log square root link function

As explained, log link and square root link functions are suitable to link a Poisson response on input variables of multistage processes. In this paper, by combining these two link functions, a new link function, called the log square root, is proposed for the Poisson response as

$$w(\mu) = \log\left(\sqrt{\mu}\right). \tag{6}$$

The mean of the Poisson response (λ) is obtained by the inverse of the link function as

$$\lambda = e^{2\left(\mathbf{x}'\boldsymbol{\beta}\right)}. \tag{7}$$

As a result, the Poisson response variable, y, can be expressed by

$$y \sim \text{Poisson}(\lambda). \tag{8}$$

3.2 Standardized residual

The standardized residuals (SRs) of the Poisson regression model are obtained to be used as a statistic of the control charts developed later. The standardized residual is defined as

$$SR_j = \frac{y_j - \lambda_j}{\sqrt{\lambda_j}}, \tag{9}$$

where y_j is jth observation of the Poisson response variable. Replacing λ_j by $e^{2\left(\mathbf{x}'\boldsymbol{\beta}\right)}$ in Eq. 7, the following statistic is obtained:

$$SR_j = \frac{y_j - e^{2\left(\mathbf{x}_j'\boldsymbol{\beta}\right)}}{\sqrt{e^{2\left(\mathbf{x}_j'\boldsymbol{\beta}\right)}}}. \tag{10}$$

The statistic in Eq. 10 will be used to develop the proposed control chart in Section 3.4.

3.3 Distribution of the standardized residuals

Two normality tests on the SR statistic are performed in this section to investigate whether it follows a normal or a non-normal distribution. The first normality test is performed based on 1,000 simulated Poisson responses, where the quality characteristic of the first stage is normal with the mean 3 and variance 1. Also, the second normality test is performed, where the quality characteristic of first stage follows Poisson distribution with the mean 2. The normal probability plots of SR are shown in Figs. 1 and 2. The plots show that SR approximately follows a standard normal distribution. Moreover, the p value of the Anderson-Darling (AD) test statistic is 0.918 and 0.901 in two tests, respectively. These results confirm the standard normal distribution of SR when the quality characteristic of first stage is normal or Poisson

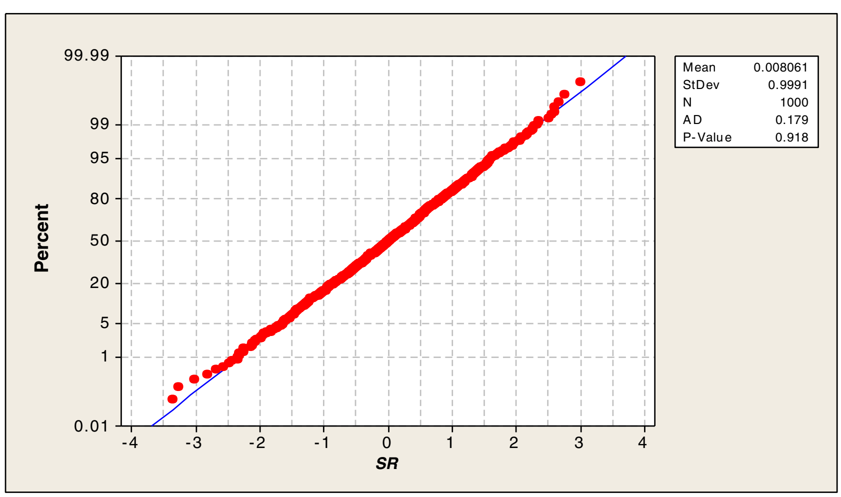

Fig. 1 Normal probability plot of SR based on log square root link function when the quality characteristic of the first stage is normal

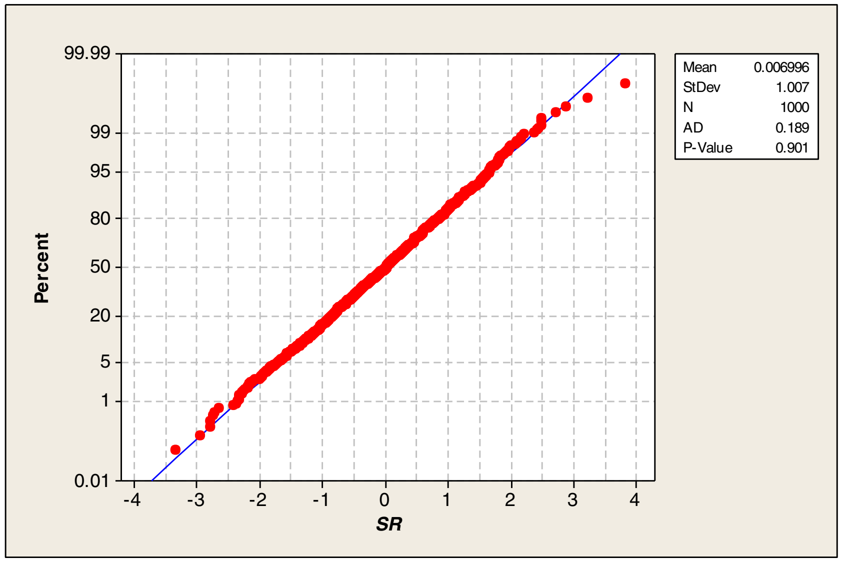

Fig. 2 Normal probability plot of *SR* based on log square root link function when the quality characteristic of the first stage is Poisson

distribution. Further, the regression parameters of the first normality test, namely β_0 and β_1, are considered 3 and 2, respectively. Also, these parameters are considered 2 and 2 in the second normality test, respectively.

3.4 Proposed control charts

Two control charts, one based on a Shewhart and the other based on an EWMA control chart, are proposed in this section to monitor the standardized residuals of Poisson response variables given in Eq. 10. The upper and lower control limits as well as the center line of the Shewhart-*SR* control chart are defined as

$$\begin{aligned} UCL &= \overline{SR} + k\sqrt{s^2(SR)}, \\ CL &= \overline{SR}, \\ LCL &= \overline{SR} - k\sqrt{s^2(SR)}, \end{aligned} \tag{11}$$

where $\overline{SR}$ and $s^2(SR)$ are the sample mean and sample variance of the *SR*s obtained by simulation, respectively, and k is the control limit coefficient determined such that a desired in-control average run length (ARL_0) is obtained. Note that the standardized residuals are drawn on this control chart as statistics.

In addition, one can monitor the mean of the residuals by an EWMA control chart. The control limits and the center line for the EWMA-*SR* control chart are computed by

$$\begin{aligned} UCL &= \overline{SR} + L\sqrt{\left(\frac{\lambda}{2-\lambda}\right)s^2(SR)}, \\ CL &= \overline{SR}, \\ LCL &= \overline{SR} - L\sqrt{\left(\frac{\lambda}{2-\lambda}\right)s^2(SR)}, \end{aligned} \tag{12}$$

where λ is the smoothing parameter that is commonly considered 0.2 and L is the control limit coefficient obtained by simulation to have a desired ARL_0. The statistic in the EWMA-*SR* control chart is as follows:

$$w_j = \lambda\, SR_j + (1-\lambda)w_{j-1} \quad ; \quad j = 1, \ldots, n. \tag{13}$$

4 Simulation studies

In this section, simulation experiments on two examples are performed to evaluate and compare the performances of the proposed schemes to the ones obtained by the method of Skinner et al. [18]. In example 1, we consider a two-stage process with normal quality characteristic in the first stage, and in example 2, we consider a Poisson (instead of normal) quality characteristic for the first stage. Also, the quality characteristic of the second stage in both examples is Poisson-distributed.

4.1 Example 1

Suppose, in an industrial workshop, there is a process with two dependent stages. The quality characteristic (x) of the first stage is normally distributed with mean 3 and variance 1. The quality characteristic of the second stage is Poisson with mean λ obtained by Eq. 7. The mean of the Poisson response is related to the quality characteristic of the first stage by the link function proposed in Section 3.1 and the link function used by Skinner et al. [18]. The regression parameters (β_0, β_1) have been obtained in phase I monitoring as 3 and 2, respectively.

The simulation experiments are performed as follows:

1. Firstly, 1,000 observations on normal input variable (x) are generated.
2. The mean of the Poisson response variable (λ) is determined for each observation by Eq. 7.
3. The Poisson response variable is generated for each observation generated in the first step by Eq. 8.
4. The standardized residual (*SR*) for each observation is obtained by Eq. 9.
5. The control limits of each control chart (Shewhart-*SR* and EWMA-*SR*) are determined to achieve ARL_0 of 200. As a result, the upper and lower control limits of the Shewhart-*SR* control chart are 2.8071 and −2.8071, respectively. Moreover, the upper and lower control limits of the EWMA-*SR* control chart are +0.8785 and −0.8785, respectively.
6. Finally, the simulation is repeated 10,000 times under three types of shift, and the comparisons between the proposed schemes and the monitoring method of Skinner et al. [18] are performed in terms of the *ARL* criterion. The shifts are as follows:

   (a) Additive and ablative shifts in the mean of the response variable through the change in β_0 or β_1 as $\beta_0 \pm \delta$ and $\beta_1 \pm \delta$.
   (b) Simultaneous shifts in β_0 and β_1 but in different directions. For example, β_0 shifts to $\beta_0 + \lambda$ and simultaneously β_1 changes to $\beta_1 - \lambda$.
   (c) Additive or ablative shifts in the mean of the incoming variable (μ_x)

4.1.1 Changes in the intercept from β_0 to $\beta_0 \pm \delta$

The results of the simulation experiments in which the intercept parameter β_0 is shifted to $\beta_0 \pm \delta$ are summarized in Table 1 for all the schemes, where r represents the Skinner et al. [18] procedure and *SR* shows the proposed method.

As shown in Table 1, the proposed method outperforms the procedure of Skinner at al. [18] under all shifts considered in both Shewhart and EWMA control charts. Moreover, the Shewhart-*SR* control chart performs uniformly better than the EWMA-*SR* scheme under this type of shift.

4.1.2 Shift in β_1

The results of the simulation experiments in which the slope parameter β_1 is shifted to $\beta_1 \pm \delta$ are summarized in Table 2 for all the schemes. The results in Table 2 show that the proposed method has better performances than the r method of Skinner et al. [18]. Once again, the Shewhart-*SR* control chart performs better than the EWMA-*SR* chart in all shifts considered.

4.1.3 Simultaneous shifts in β_0 and β_1

In this subsection, simultaneous positive and negative shifts in β_0 and β_1 are considered, where the simulation results are reported in Table 3.

The results in Table 3 illustrate that the *SR* method has better performances than the r method under simultaneous shifts in the regression parameters. Further, the Shewhart-*SR* chart outperforms EWMA-*SR* scheme in all shifts considered.

4.1.4 Shift in μ_x

The main purpose of a cause-selecting control chart in multi-stage processes is to eliminate the effect of the previous stages on the characteristics of the current stage. Since this chart is used to monitor the quality characteristic of the second stage of two-stage processes, changes in the mean of the quality characteristic of the first stage, μ_x, should not affect the statistic used in the second stage of monitoring the response variable. In this subsection, μ_x is changed to $\mu_x \pm \nu$, and then

Table 1 The *ARL* performance (and standard error of *ARL*) under shift in β_0 in example 1

δ	$\beta_0+\delta$				$\beta_0-\delta$			
	Shewhart		EWMA		Shewhart		EWMA	
	r	*SR*	r	*SR*	r	*SR*	r	*SR*
0	200.4(2)	200.1(2)	200.1(2)	200.2(2)	200.4(2)	200.1(2)	200.1(2)	200.2(2)
0.0000005	200.2(1.9)	131.9(1.3)	199.4(1.8)	145.7(1.4)	200.3(1.9)	136.2(1.3)	200.8(1.9)	149.5(1.5)
0.000001	199.8(1.8)	81.5(0.8)	199.3(1.8)	94.4(0.9)	199.7(1.8)	83.7(0.8)	198.4(1.9)	98.2(0.9)
0.0000025	199.1(1.8)	39.3(0.4)	198.3(1.8)	44.5(0.4)	199.3(1.8)	37.5(0.4)	198(1.8)	42.2(0.4)
0.000005	198.9(1.8)	21.6(0.2)	198.2(1.8)	23.1(0.2)	198.7(1.8)	20.6(0.2)	197.8(1.8)	20.8(0.2)
0.00001	198.4(1.8)	11(0.1)	197.4(1.8)	12.2(0.1)	198.2(1.8)	10.7(0.1)	196.6(1.8)	11.5(0.1)
0.00005	197.1(1.8)	3.5(0.03)	197.9(1.8)	3.7(0.04)	197.3(1.8)	3.3(0.03)	195.8(1.8)	3.8(0.04)
0.0005	188.3(1.7)	1.4(0.01)	168.5(1.6)	1.5(0.01)	186.3(1.7)	1.4(0.01)	166.4(1.6)	1.5(0.01)

Table 2 The *ARL* performance (and standard error of *ARL*) under shift in β_1 in example 1

δ	$\beta_1+\delta$				$\beta_1-\delta$			
	Shewhart		EWMA		Shewhart		EWMA	
	r	*SR*	*r*	*SR*	*r*	*SR*	*r*	*SR*
0	200.4(2)	200.1(2)	200.1(2)	200.2(2)	200.4(2)	200.1(2)	200.1(2)	200.2(2)
0.00000005	200.1(1.9)	163(1.6)	191.5(1.8)	173.7(1.7)	200.1(1.9)	167.4(1.7)	193.4(1.9)	171.5(1.7)
0.0000001	199.6(1.8)	127.5(1.3)	189.2(1.8)	132.2(1.3)	199.5(1.8)	129.3(1.3)	192.7(1.8)	139.5(1.4)
0.00000015	198.8(1.8)	100.9(1)	188.7(1.8)	109.2(1.1)	199.4(1.8)	101.5(1)	190.1(1.8)	108.6(1.1)
0.0000003	198.5(1.8)	62.6(0.6)	188.7(1.8)	67.8(0.7)	198.9(1.8)	59.7(0.6)	189.1(1.8)	67.1(0.6)
0.000001	198.1(1.8)	21.7(0.2)	188.6(1.8)	22.9(0.2)	198.4(1.8)	22.1(0.2)	188.4(1.8)	23.1(0.2)
0.00001	197.5(1.8)	4.2(0.04)	186.9(1.8)	4.6(0.04)	1.97.2(1.8)	4.2(0.04)	187.8(1.8)	4.7(0.04)
0.0001	189.3(1.7)	1.6(0.01)	173.6(1.7)	1.8(0.01)	187.3(1.7)	1.6(0.01)	176.8(1.7)	1.8(0.01)

the effect of the change on the performances of all charts is evaluated. The results of the simulation experiments are presented in Table 4.

Based on Table 4, it is clear that the *ARL* of Shewhart-*SR* and EWMA-*SR* control charts are not affected under both additive and ablative shifts in μ_x. However, the *ARL* of Shewhart-*r* and EWMA-*r* control charts are changed under negative large shifts.

In summary, based on the results in Tables 1, 2, 3, and 4, one can conclude that the proposed *SR* method is better than the *r* method to monitor two-stage processes with a Poisson response variable.

4.2 Example 2

In this example, we consider the Poisson distribution with mean 2 for the quality characteristic of the first stage. Also, the quality characteristic of the second stage is Poisson with mean λ obtained by Eq. 7. The regression parameters β_0 and β_1 are obtained based on phase I analysis as 2 and 2, respectively.

The simulation and comparison between the proposed schemes and the monitoring method of Skinner et al. [18] are performed based on three types of shift similar to the presented example in the Section 4.1. Note that the quality characteristic of the first stage (x) in this example follows Poisson distribution. Hence, each observation of x is generated based on Poisson distribution in this section.

As a result, the upper and lower control limits of the Shewhart-*SR* control chart in example 2 are 2.8077and −2.8077, respectively. Moreover, the upper and lower control limits of the EWMA-*SR* control chart are +0.8783 and −0.8783, respectively.

4.2.1 Shift in β_0

The results of the simulation experiments in which the intercept parameter β_0 is shifted to $\beta_0 \pm \delta$ are summarized in Table 5. Similar to the results obtained in Section 4.1.1, the proposed method outperforms the procedure by Skinner et al. [18] under all shifts considered in both Shewhart and EWMA control charts. Moreover, the Shewhart-*SR* control chart performs better than the EWMA-*SR* chart in all shifts considered.

Table 3 The *ARL* performance (and standard error of *ARL*) under simultaneous shifts in β_0 and β_1 in example 1

λ	$\beta_0+\lambda$ and $\beta_1-\lambda$				$\beta_0-\lambda$ and $\beta_1+\lambda$			
	Shewhart		EWMA		Shewhart		EWMA	
	r	*SR*	*r*	*SR*	*r*	*SR*	*r*	*SR*
0	200.4(2)	200.1(2)	200.1(2)	200.2(2)	200.4(2)	200.1(2)	200.1(2)	200.2(2)
0.0000005	200.2(1.9)	42.2(0.5)	199.8(1.9)	53.34(0.5)	200.3(1.9)	44.47(0.5)	197.9(1.9)	54.2(0.5)
0.000001	200.1(1.9)	27.7(0.3)	199.2(1.9)	30.6(0.3)	199.9(1.9)	25.5(0.3)	193.5(1.8)	29.3(0.3)
0.000005	198.3(1.8)	7.1(0.07)	197.4(1.8)	8.9(0.08)	197.8(1.8)	7.9(0.08)	192.7(1.8)	8.4(0.08)
0.00005	194.9(1.8)	2.4(0.02)	194.2(1.8)	2.7(0.02)	192.8(1.8)	2.4(0.02)	191.8(1.8)	2.5(0.02)
0.0005	72(0.7)	1.3(0.01)	63(0.6)	1.4(0.01)	76(0.7)	1.3(0.01)	61.8(0.6)	1.4(0.01)

Table 4 *ARL* (and standard error of *ARL*) performance under shifts of size $\pm\nu$ in μ_x in example 1

ν	$\mu_x+\nu$				$\mu_x-\nu$			
	Shewhart		EWMA		Shewhart		EWMA	
	r	*SR*	*r*	*SR*	*r*	*SR*	*r*	*SR*
0	200.4(2)	200.1(2)	200.1(2)	200.2(2)	200.4(2)	200.1(2)	200.1(2)	200.2(2)
0.1	200.4(1.9)	199.9(1.9)	199.2(1.9)	202.5(1.9)	198.3(1.9)	202.3(1.9)	199.2(1.8)	201(1.9)
0.25	196.6(1.9)	199.7(2)	199.6(1.9)	202.7(2.1)	198.8(1.9)	199.2(1.9)	199(2.1)	200.6(2)
0.5	201.4(1.9)	199.7(1.9)	201.5(2)	200.9(2)	200.2(2)	202(2)	198.3(1.7)	199.4(1.9)
0.75	204.6(2.1)	199.7(1.9)	201.9(2.1)	197.1(1.9)	192.1(1.9)	201.7(1.9)	195.1(1.8)	200.1(2)
1	200.6(2)	201(1.9)	201(2)	195.4(1.8)	188(1.8)	198.4(1.9)	188.1(1.7)	194.1(1.9)
1.5	200.8(1.9)	199.1(1.9)	197.2(1.9)	205.1(2)	147.1(1.5)	199.9(1.9)	150.7(1.4)	199.2(1.9)
2	195.2(1.9)	201.7(2)	197.9(1.9)	200.1(2)	85.7(0.9)	201.9(2)	86.2(0.8)	204.5(2)

4.2.2 Changes in the slope from β_1 to $\beta_1 \pm \delta$

The results of the simulation runs in which the slope parameter β_1 is shifted to $\beta_1 \pm \delta$ are summarized in Table 6 for all the schemes.

The results in Table 6 show that the proposed method has better performances than the *r* method of Skinner et al. [18], and also, the Shewhart-*SR* control chart performs better than the EWMA-*SR* chart in all shifts considered.

4.2.3 Simultaneous shifts in intercept and slope parameters

In this subsection, simultaneous additive and ablative shifts in β_0 and β_1 are considered, and the simulation results are gathered in Table 7.

The results in Table 3 show that the *SR* method has better performance than the *r* method under simultaneous shifts in the regression parameters. In addition, the Shewhart-*SR* chart outperforms the EWMA-*SR* scheme in all shifts considered.

4.2.4 Shift in μ_x

In this subsection, we change μ_x to $\mu_x \pm \nu$ and then evaluate the effect of change on the performance of both Shewhart and EWMA (*r* and *SR*) control charts. The results of the simulation are presented in Table 8.

Based on Table 7, it is clear that the *ARL* of Shewhart-*SR* and EWMA-*SR* control charts are not affected under both additive and ablative shifts in μ_x. However, the *ARL* of Shewhart-*r* and EWMA-*r* control charts are changed under negative large shifts.

Based on the results of Tables 5, 6, 7, and 8, we conclude that the proposed *SR* method is better than the *r* method for monitoring two-stage processes with a Poisson response variable.

Generally, based on the two considered examples, the proposed methods and the method of Skinner et al. [18] are not very dependent on the distribution type of the quality characteristic of the first stage, but the method of Skinner et al. [18] is more dependent on the distribution of the quality

Table 5 The *ARL* performance (and standard error of *ARL*) under shift in β_0 in example 2

δ	$\beta_0+\delta$				$\beta_0-\delta$			
	Shewhart		EWMA		Shewhart		EWMA	
	r	*SR*	*r*	*SR*	*r*	*SR*	*r*	*SR*
0	200.3(2)	200.8(2)	200.1(2)	200.3(2)	200.3(2)	200.8(2)	200.1(2)	200.3(2)
0.0000005	199.6(1.9)	98(1)	199.7(1.9)	103.4(1)	199.6(1.9)	97.2(1)	196.7(1.9)	106.5(1)
0.000001	199.5(1.9)	75.4(0.7)	199.4(1.9)	87.9(0.8)	199.5(1.9)	71(0.7)	196.7(1.9)	87.1(0.8)
0.0000025	199.3(1.9)	46.4(0.4)	197.2(1.9)	48.2(0.4)	198.3(1.9)	43.8(0.4)	196.3(1.9)	47.5(0.4)
0.00001	197.2(1.9)	21.7(0.2)	195.8(1.9)	28.2(0.3)	197.2(1.9)	23(0.2)	195.3(1.9)	28.3(0.3)
0.00005	196.5(1.9)	12.1(0.1)	195(1.9)	13.3(0.1)	196.5(1.9)	12.7(0.1)	194.8(1.9)	13.7(0.1)
0.0005	162.7(1.6)	4.1(0.04)	158.1(1.5)	4.8(0.04)	167.7(1.8)	4.1(0.04)	159.7(1.5)	4.6(0.04)
0.005	33.3(0.3)	1.7(0.01)	33(0.3)	2.1(0.02)	34.4(0.3)	1.7(0.01)	32.5(0.3)	2.1(0.02)

Table 6 The *ARL* performance (and standard error of *ARL*) under shift in β_1 in example 2

δ	$\beta_1+\delta$				$\beta_1-\delta$			
	Shewhart		EWMA		Shewhart		EWMA	
	r	*SR*	*r*	*SR*	*r*	*SR*	*r*	*SR*
0	200.3(2)	200.8(2)	200.1(2)	200.3(2)	200.3(2)	200.8(2)	200.1(2)	200.3(2)
0.00000005	197.6(1.9)	99.3(1)	196.9(1.9)	112.7(1.1)	198.2(1.9)	95.7(1)	196.4(1.9)	102(1)
0.00000015	197.5(1.9)	81.5(0.8)	197.4(1.9)	89.6(0.9)	198.5(1.9)	81.7(0.8)	196.2(1.9)	92.7(0.9)
0.0000003	196.7(1.9)	48.9(0.5)	197.6(1.9)	63.2(0.6)	197.4(1.9)	48(0.5)	197.3(1.9)	58.9(0.6)
0.000001	192.9(1.9)	37.7(0.4)	194.7(1.9)	39.7(0.4)	193.7(1.8)	38.5(0.4)	197.3(1.9)	40.7(0.4)
0.00001	188.1(1.9)	13.6(0.1)	191.3(1.9)	14.8(0.1)	185.7(1.8)	14.6(0.1)	191.4(1.9)	15.2(0.1)
0.0001	146(1.4)	5.6(0.05)	150.8(1.5)	5.8(0.05)	139.6(1.4)	5.5(0.05)	141.6(1.5)	6.1(0.05)
0.01	5.9(0.05)	1.6(0.01)	6(0.05)	1.6(0.01)	5.7(0.05)	1.6(0.01)	6(0.05)	1.7(0.01)

characteristic of the first stage rather than the proposed method.

5 Case study: Etch process in semiconductor wafer

In this section, the case study in Skinner et al. [18] is used to investigate the performance of the proposed method. In this case, the mean numbers of defects of two radii on the wafer are measured after the etch process by inter-digitised comb and serpentine on the semiconductor wafer. In this process, in addition to counting defects at the end of the process, the various in-process variables, such as gas flow, temperature, and exhaust, are measured. Skinner et al. [18] assumed that the mean number of defects of the radii can be related to the exhaust flow by the GLM regression. In this regard, the quality characteristics of the number of defects on the internal and external radii are considered as quality characteristics of the second stage (output variables). Also, the exhaust flow is considered as quality characteristic of the first stage (input variable). Hence, the output variables (Y_1 and Y_2) follow a Poisson distribution, and the input variable (X) follows a uniform distribution.

In practice, parameters of the GLM model are estimated using historical data; however, since Skinner et al. [18] used the simulation data for estimation purposes, we use simulation data as well. It should be noted that Skinner et al. [18] assumed the input variable (X) follows the uniform distribution over [0, 3] and the output variables (Y_1 and Y_2) follow the Poisson distribution with $\lambda_1=\exp(0.5+1.0X)$ and $\lambda_2=\exp(0.25+1.25X)$, respectively.

In the simulation study, 1000 input variables are first generated based on a uniform distribution. Then, the Poisson output variables based on the proposed link function are generated by simulation. Since, the input variable is assumed normal in this paper, the uniform variable is transformed to a normal variable using the NORTA Inverse method. Then, we estimated the parameters (β_0 and β_1) of the GLM regression model based on the proposed link function using the transformed input data (normal input data) and each of Poisson output data. The estimated parameters ($\widehat{\beta}_{0k}$ and $\widehat{\beta}_{1k}$; $k=1, 2$) of the GLM regression model based on the proposed link

Table 7 The *ARL* performance (and standard error of *ARL*) under simultaneous shifts in β_0 and β_1 in example 2

λ	$\beta_0+\lambda$ and $\beta_1-\lambda$				$\beta_0-\lambda$ and $\beta_1+\lambda$			
	Shewhart		EWMA		Shewhart		EWMA	
	r	*SR*	*r*	*SR*	*r*	*SR*	*r*	*SR*
0	200.3(2)	200.8(2)	200.1(2)	200.3(2)	200.3(2)	200.8(2)	200.1(2)	200.3(2)
0.0000005	185.5(1.8)	44.5(0.4)	198(1.9)	49.6(0.5)	199.4(2)	48.1(0.5)	200.1(2)	50.5(0.5)
0.000005	197.1(2)	17.5(0.1)	194.3(1.8)	17.9(0.1)	198.2(2.1)	18.1(0.1)	199(2)	18.2(0.1)
0.00005	175.6(1.8)	6.9(0.07)	180.7(1.8)	7(0.07)	181.6(1.8)	6.7(0.07)	181.7(1.9)	7(0.07)
0.0005	63.7(0.6)	3.1(0.03)	73.6(0.7)	3.1(0.03)	61.7(0.6)	3.1(0.03)	75.3(0.7)	3.1(0.03)
0.005	12.2(0.1)	1.7(0.01)	13.2(0.1)	2.1(0.02)	11.3(0.1)	1.7(0.01)	12.8(0.1)	2.1(0.02)

Table 8 *ARL* (and standard error of *ARL*) performance under shifts of size $\pm\nu$ in μ_x in example 2

ν	$\mu_x+\nu$				$\mu_x-\nu$			
	Shewhart		EWMA		Shewhart		EWMA	
	r	*SR*	*r*	*SR*	*r*	*SR*	*r*	*SR*
0	200.3(2)	200.8(2)	200.1(2)	200.3(2)	200.3(2)	200.8(2)	200.1(2)	200.3(2)
0.1	201.9(1.9)	200.1(1.9)	199.7(1.9)	201.3(2)	201.3(1.9)	199.7(1.9)	201.2(1.9)	198.4(1.9)
0.25	199.5(1.9)	200.2(1.9)	198.7(1.9)	200.5(2)	200.8(1.9)	2002(2)	198.4(1.9)	202.6(2)
0.5	200.4(1.9)	200.9(2)	202.9(2)	200.9(2)	199.2(2)	202(2)	197.3(1.9)	199.7(1.9)
0.75	198.2(2.1)	199.3(1.9)	201.2(2)	199.1(1.9)	195.1(1.9)	199.4(1.9)	193.2(1.9)	199.8(1.9)
1	200.1(2)	199.5(1.9)	199(1.9)	205.4(2.1)	189.7(1.8)	199.8(1.9)	189.4(1.8)	198.3(1.9)
1.5	201.8(2)	200.1(1.9)	198.2(1.9)	199.1(1.9)	160.6(1.5)	199.9(1.9)	184.7(184)	196.2(1.9)
2	205.2(2)	199.7(2)	202.6(1.9)	198.1(1.9)	142.4(1.4)	201.6(2)	169.3(1.6)	203.4(2)

function and the log link function, used by Skinner et al. [18], are shown in Table 9.

Note that two independent output variables (Y_1 and Y_2) follow Poisson distribution with λ_1 and λ_2 parameters, respectively. Also, the input variable (X) follows normal distribution with $\mu_X = 1.5$ and $\sigma_x = \sqrt{0.75}$ after transformation by NORTA inverse technique. So, based on the results of Table 9, when the second stage is in-control, λ_1 and λ_2 for the proposed link function in the proposed statistic (*SR*) are as follows:

$$\lambda_1 = e^{2(1.252 + 0.75X)}. \tag{14}$$

$$\lambda_2 = e^{2(1+1X)}. \tag{15}$$

Also, the corresponding λ_1 and λ_2 for the log link function (Eq. 2) are

$$\lambda_1 = e^{(1.06+0.79X)} \tag{16}$$

and

$$\lambda_2 = e^{(0.886 + 1X)}. \tag{17}$$

Table 9 The estimated parameters of the proposed and logarithm link functions for two output variables

Kind of link function	Output variable	$\widehat{\beta}_0$	$\widehat{\beta}_1$
Proposed link function	Y_1	1.252	0.75
	Y_2	1	1
Logarithm link function	Y_1	1.06	0.79
	Y_2	0.886	1

The proposed statistic (*SR*) and deviance residual (*r*) for each data are obtained by Eqs. 10 and 3, respectively. Then, the means of *SR* ($\overline{SR}$) and *r* ($\overline{r}$) as *CL*s are calculated. Finally, the *UCL* and *LCL* of the Shewhart and the EWMA schemes for the *SR* and *r* statistics are obtained by simulation through 10,000 runs to achieve the desired in-control *ARL* equal of 200. The results are presented in Table 10.

Twenty in-control observations (j=1,…,20) are first generated for each statistic in each control chart and then out-of-control observations are generated to observe out-of control signals in each control chart. The results are illustrated in Figs. 3, 4, 5, 6, 7, 8, 9, and 10. In these figures, the EWMA-*SR* and EWMA-*r* statistics are denoted by SR_kW (k=1, 2) and r_kW (k=1, 2), respectively, and k shows the first and second Poisson response variables.

Figures 3 and 4 show that the Shewhart-*SR* control chart has better performance than the Shewhart-*r* control chart because when we generated out-of-control Y_1 data, the Shewhart-*SR* chart detected the out-of-control status in the second out-of-control data or j=22th observation. However,

Table 10 The simulated *CL*, *UCL*, and *LCL* of Shewhart and EWMA control charts

Kind of control chart	Output variable	*UCL*		*CL*		*LCL*	
		SR	*r*	*SR*	*r*	*SR*	*r*
Shewhart	Y_1	2.78	2.562	−0.072	−0.517	−2.924	−3.596
	Y_2	2.887	2.571	0.0675	−0.428	−2.752	−3.428
EWMA	Y_1	0.823	0.733	−0.072	−0.517	−0.967	−1.766
	Y_2	0.959	0.735	0.0675	−0.428	−0.824	−1.591

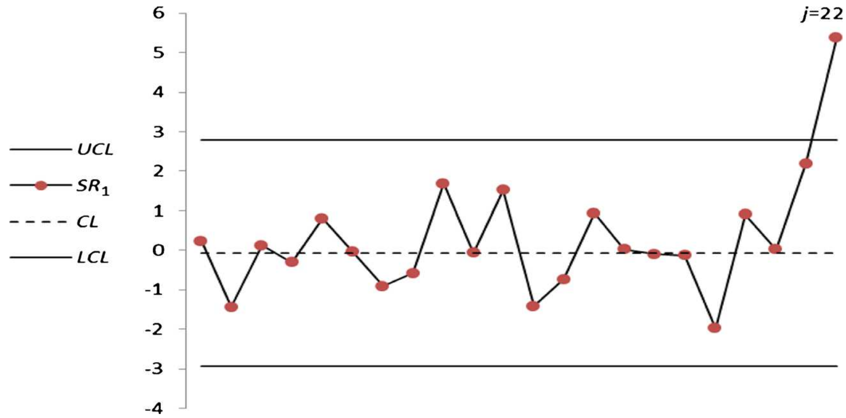

Fig. 3 The Shewhart-SR_1 control chart for monitoring the proposed statistic when β_{01} = 1.252 shifts to β_{01} = 1.252 + 0.05 after the 20th observation in the first output variable (Y_1)

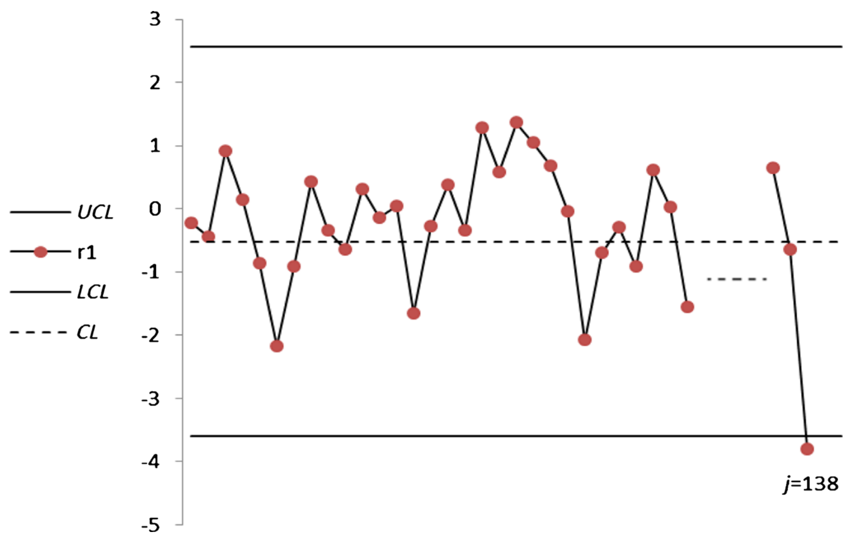

Fig. 4 The Shewhart-r_1 control chart for monitoring the r statistic when β_{01} = 1.06 shifts to β_{01} = 1.06 + 0.05 after the 20th observation in the first output variable (Y_1)

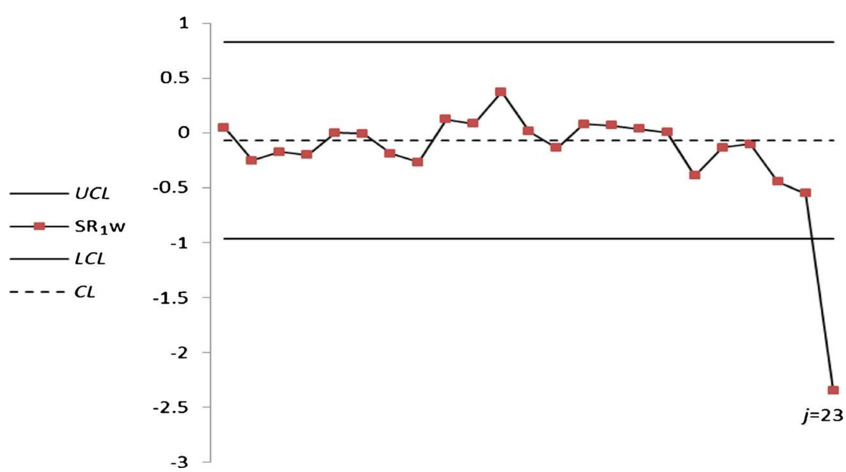

Fig. 5 The EWMA-SR_1 control chart for monitoring the proposed statistic when β_{11} = 0.75 shifts to β_{11}=1.06 − 0.05 after the 20th observation in the first output variable (Y_1)

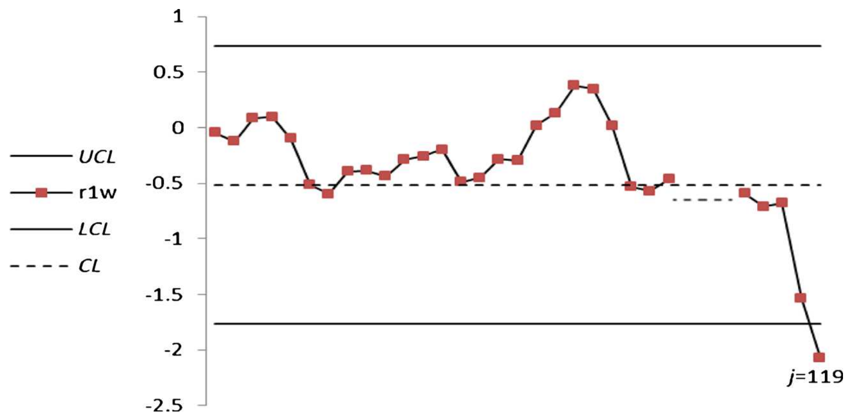

Fig. 6 The EWMA-r_1 control chat for monitoring the r statistic when $\beta_{11} = 0.79$ shifts to $\beta_{11} = 0.79 - 0.05$ after the 20th observation in the first output variable (Y_1)

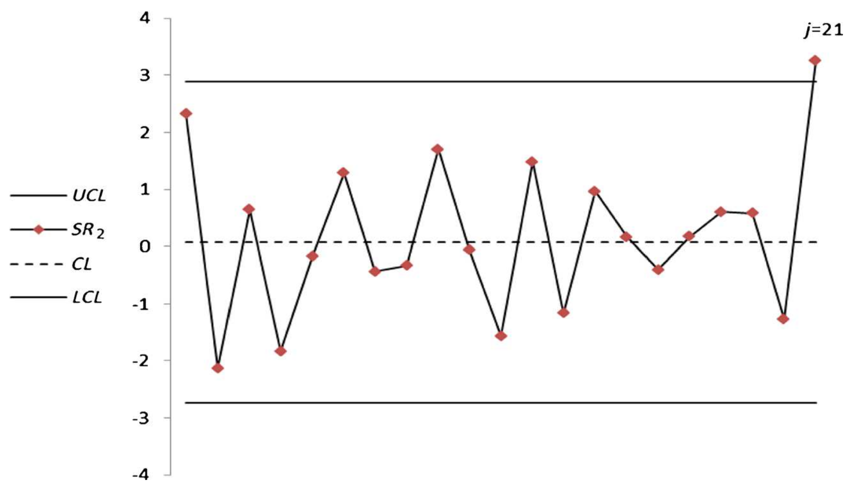

Fig. 7 The Shewhart-SR_2 control chart for monitoring the proposed statistic when $\beta_{12} = 1$ shifts to $\beta_{12} = 1 + 0.05$ after the 20th observation in the second output variable (Y_2)

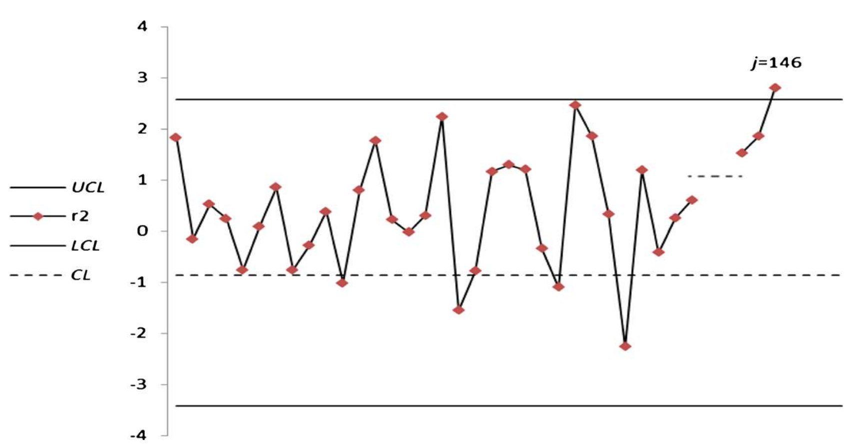

Fig. 8 The Shewhart-r_2 control chart for monitoring the r statistic when $\beta_{12} = 1$ shifts to $\beta_{12} = 1 +$ 0.05 after the 20th observation in the first output variable (Y_2)

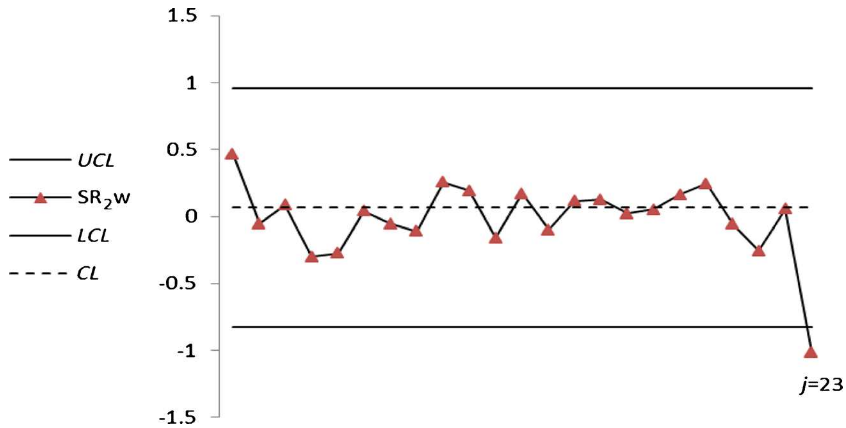

Fig. 9 The EWMA-SR_2 control chart for monitoring the proposed statistic when β_{02} = 1 shifts to β_{02} = 1 − 0.05 after the 20th observation in the second output variable (Y_2)

the Shewhart-r signaled the out-of-control alarm in j=138th observation.

Moreover, Figs. 5 and 6 show that the EWMA-SR_1 control chart has better performance than the Shewhart-r_1 control chart because it detects the out-of-control state more quickly.

Figures 7 and 8 show that the Shewhart-SR_2 control chart has better performance than the Shewhart-r_2 control chart because it detects more quickly the out-of-control state.

The results of Figs. 9 and 10 show that the EWMA-SR_2 control chart has better performance in comparison with the EWMA-r_2 control chart.

Generally, results of Figs. 3, 4, 5, 6, 7, 8, 9, and 10 show that performance of the proposed link function and chart is better than the one of Skinner et al. [18] method. Hence, to monitor a two-stage processes with Poisson quality characteristic in the second stage, we recommend using the proposed method along with the proposed link function.

6 Conclusion and future researches

In this paper, a new method was proposed to monitor two-stage processes with a Poisson variable in the second stage. To do this, the logarithm and the square root link functions were first combined in a new link function to establish a relationship between the Poisson response and the input variables. Then, instead of the deviance residuals used in the r method of Skinner et al. [18], a standardized residual (SR) statistic was obtained. This SR statistic is independent of the quality characteristic in the previous stage. Through simulation experiments, we showed that the SR statistic is approximately standard normal. Afterwards, EWMA-SR and Shewhart-SR control charts were developed to monitor the SR statistic. Finally, the performance of the proposed method was evaluated through two examples in terms of the average run length (ARL) criterion in comparison with the r method. The comparisons that were made based on three types of shifts showed

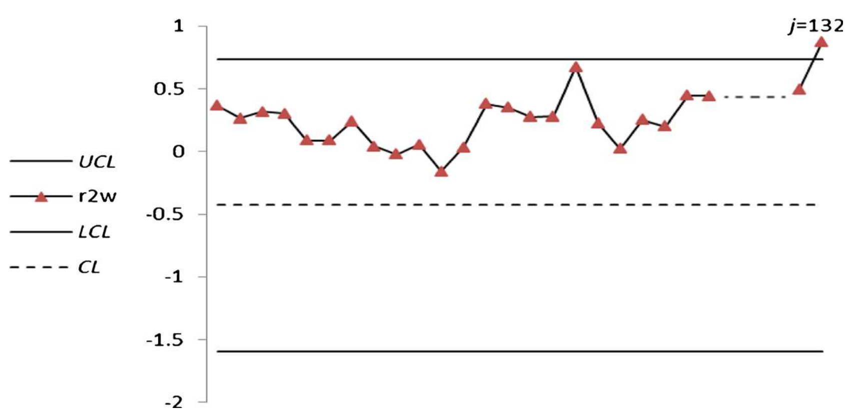

Fig. 10 The EWMA-r_2 control chat for monitoring the r statistic when β_{02} = 0.886 shifts to β_{02} = 0.886 − 0.05 after the 20th observation in the second output variable (Y_2)

that the proposed *SR* method outperforms the *r* method in all changes. Also, we considered the case study by Skinner et al. [18] and showed that the proposed method has better performance than the *r* method by Skinner et al. [18]. We suggest the economic and economic-statistical design of the proposed control chart as future researches. Also, one can consider outliers in historical data and propose the robust method to account for this problem.

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