Joint Optimization of Statistical Process Control, Preventive Maintenance and Production Scheduling

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Abstract: In real world, production, maintenance and process quality have strong relationship with each other, however, most of the papers take into consideration these aspects as separate problems. Information obtained from signals given by a control chart affect on maintenance actions. Correspondingly, maintenance activities have a direct impact on equipment availability and product quality characteristics. Also, their results influence on the performance of production systems, hence they should be jointly optimized. This paper provides an optimal sequence with considering optimal preventive maintenance interval and control chart parameters where total expected cost per unit time is minimized. A numerical example is given to illustrate the proposed methodology.

Keywords: Joint optimization, Production scheduling, Statistical process control, Preventive maintenance, Optimal sequence.

1. Introduction

In today's competitive growing global market, it is essential to consider interrelationship between quality control, maintenance planning and production scheduling with integrated management. There are extensive researches on combining maintenance and statistical process control (SPC). Tagaras [1] investigated SPC and preventive maintenance (PM) simultaneously and optimized the design parameters under a Markov chain deterioration assumption. Ho and Quinino [2] proposed a model that integrates SPC and corrective maintenance (CM) and used a geometric distribution to describe shifts from in-control to out-of-control states. Similarly there are many papers that have been investigated the relationship between SPC, and production scheduling (PS), such as Dhouib et al. [3]. They utilized an integrated approach for joint optimization of production–inventory control and PM policy. Despite some literature is available for integrating maintenance with scheduling and maintenance with statistical process control, integration of all of them has been made a few attention in papers. Rahim et al. [4] proposed an integrated model including SPC, PS and maintenance to obtain optimal maintenance level and enhance the quality of process. In this paper, we propose joint optimization of the SPC, PS, and PM to find the optimal location of PM and minimize the total cost. The structure of the paper is as follows: In section 2, the problem and the corresponding parameters are defined. In section 3, our proposed method is explained. A numerical example is illustrated in section 4.Our concluding remarks as well as future researches are given in the final section.

2. Statement of the Problem

This research has been organized based on the research of Xiang [5] .In addition of his work we consider joint optimization of production scheduling with SPC and PM policy. First we joint SPC and maintenance with methodology similar to Xiang [5]. Then, we use optimal PM interval and control chart parameters to minimize expected total cost. In this method, optimal PS is obtained with considering location of the PM.

To determine the optimal batches sequence, an algorithm is employed. Moreover, our solution minimizes penalty-cost incurred due to schedule delay. This paper considers additional assumptions including

1. There are m batches and m locations for PM.
2. Each period of time involves q sequences.
3. The machine cannot be stopped for PM until all the jobs in a batch are completed.

Additional parameters of the problem are as follows (Others are as the same as [5]):

New parameters:

B_i	ith Batch $i=1,2,\ldots,m$
P_i , d_i , S_i	processing time, due date and setup time of ith Batch, respectively
$Pl_{k,j}$	penalty cost of jth sequence in kth location of PM
W_i	importance of ith Batch
Cr_i	quality reduction cost of ith Batch
t_{CM} , t_{PM}	time for performing CM, and PM, respectively in any sequence
cp_j , c_j	Completion time of jth sequence with(SPC, PM and PS), and (SPC and PM), respectively.
pr, pr'	Failure probabilities of machine with and without PM actions respectively

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α, β	Probabilities of Type I and II errors
ctp_j	Total tardiness cost of jth sequence (with SPC, PM and PS)
ct_j	Total tardiness cost of jth sequence (with SPC, PM)
ct'_j, ctp'_j	Total tardiness cost of jth sequence when failure occurs for(with SPC, PM) and (with SPC, PM and PS) respectively
TCP_j	Total cost of jth sequence (with SPC, PM and PS)
TC_j	Total cost of jth sequence (with SPC, PM)

3. PROPOSED METHOD

Our methodology is based on Xiang [5] to joint SPC and PM and obtain optimal SPC parameters and PM interval. Then, we use these results to compute total cost and find the best batches sequence and the location of PM actions. In order to achieve these goals, we find optimal location of the PM for each sequence, then, determine the sequence that has minimum cost. The proposed algorithm obtains batches sequence that gives the minimum cost per unit time. In the proposed method, cost of quality reduction due to postponement of PM actions is considered as well. Total cost and tardiness cost of jth sequence is calculated by equation (1) and (2) respectively:

$$TCP_j = \frac{pr'(ctp_j + q(\frac{cp_j}{\tau})c_{PM}) + (1 - pr')(\frac{C_{CM}}{q} + t_{CM}c_{DT} + ctp'_j)}{cp_j} \tag{1}$$

$$ctp_j = \min_k \{(1-\beta)(\sum_{i=1}^{m}(T_{ikj}\,\text{Pl}_i) + \sum_{l=1}^{k} C\eta)\} + \alpha C_{ins}, k = 1, \dots, m \tag{2}$$

Penalty-cost incurred when a batch is completed after its due date and is calculated by equation (3):

$$T_{ikj} = \max\{(C_{ikj} - d_i), 0\} \tag{3}$$

Expected total cost of jth batches sequence without PM can be computed as follows:

$$TC_j = \frac{pr(ct_j) + (1 - pr)(\frac{C_{CM}}{q} + t_{CM}c_{DT} + ct'_j)}{c_j}, \tag{4}$$

where ct' is cost of tardiness while failure state is occurred and ct_j is calculated as follows:

$$ct_j = \min_k \{(1-\beta)(\sum_{i=1}^{m}(T_{ikj}\,\text{Pl}_i) + \frac{1}{m}\sum_{l=i+1}^{m} C\eta)\}, k = 1, \dots, m \tag{5}$$

In equation (5), cost of quality reduction is considered for batches which are produced after out-of-control signal by control chart in the case no PM action is done. Finally to find optimal solution, total costs are compared in any iteration and batch sequence with minimum cost is selected.

4. NUMERICAL EXAMPLE

We provide a numerical example with data extracted from [5]. Moreover, it is assumed that machine produces 3 different batches. First, we joint optimization of SPC and PM to compute optimal values of sample size, sampling interval, coefficient of $\bar{x}$ control limits and maintenance interval shown as n, h, k, τ respectively. These results are shown in Table 1.

TABLE 1
OPTIMAL RESULTS OBTAINED WITH JOINT OPTMIZATION OF SPC AND PM

Decision variable	n	h	k	τ
Optimal value	3	4	1.96	56

Then, we use these optimal values to calculate the TCP for all the possible batches sequence without PM. With considering PM in each location of batches sequence we find the best PS. For instance, in the sequence of [B1-B2-B3] if PM is performed before the first batch (B1), then the value of TCP for the given sequence is 223 (See Table 2). The sequence with the smallest total cost is determined as optimal PS sequence. (See Table3)

TABLE 2
OPTIMAL RESULTS OBTAINED WITH JOINT OPTMIZATION OF SPC AND PM

Batch sequences	Location of PM	TCP
$[PM - B_1 - B_2 - B_3]$	PM is before first batch	223

TABLE 3
COMPARISION OF COST OF JOINT SPC AND PM WITH JOINT SPC,PM AND PS

	Batch sequences	TC	TCP and location of PM	
			TCP	Location of PM
1	$[B_1 - B_2 - B_3]$	259.6	223	PM should be before B_1
2	$[B_1 - B_3 - B_2]$	607.3	**175***	**PM should be before B_2**
3	$[B_2 - B_1 - B_3]$	393	265	PM should be before B_3
4	$[B_2 - B_3 - B_1]$	595	296	PM should be before B_2
5	$[B_3 - B_1 - B_2]$	415.6	275	PM should be before B_3
6	$[B_3 - B_2 - B_1]$	611.8	338	PM should be before B_1

5. CONCLUSIONS AND FUTURE RESEARCHES

This paper optimizes SPC, PM and PS simultaneously to minimize total cost. Numerical example confirmed that this integrated solution gives an optimal PS sequence as well as minimum cost with regard to PM. Our results indicated the integrated model has the lower total cost rather than the solution without PM. Considering the number of products of each batch, process capability, and different production environment such as job shop can be interest topics for future researches.

7. REFERENCES

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