Noise Equivalent Circuit Model of Thin Avalanche Photodiodes

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***Abstract*— In this letter, a circuit model of noise is presented for thin avalanche photodiodes. By defining an effective electric field over the multiplication region and using position-dependent ionization coefficients, the model tries to include the effects of carrier's dead space and previous ionization history. Then, using a predicted multiplication gain, the model obtains the photocurrent response and output noise spectrum. The excess noise factor and spectral signal-to-noise ratio obtained from the proposed model are compared with available experimental data for different widths of multiplication region. The model is suitable for sensitivity and gain-bandwidth analysis.**

***Index Terms*— Avalanche photodiode, multiplication gain, dead space effect, excess noise factor, signal-to-noise ratio.**

I. Introduction

THIN avalanche photodiodes have generated a lot of interest in optical communication systems due to their superior noise performance. Although the built-in gain of APDs increases the system sensitivity, their noise, which mainly arises from multiplication phenomenon still significantly affects overall system SNR. For accurate modeling of thin APDs, nonlocal effects should be considered and the model must account for the path that the carriers require to travel to achieve sufficient energy and participate in impact ionization, i.e. the dead space [1], [2]. There were some efforts to include the effect of dead space and nonuniform electric field profile [3]. However, none of them can predict the noise. In this letter, we present a circuit model for thin APDs with the ability of noise estimation. In the model, nonlocal effects are treated by partially defining and modifying the electric field profile throughout the multiplication region.

II. Circuit Model Theory Review

The electric field profile of a typical PIN-APD under a reverse bias voltage is shown in Fig. 1(a). The multiplication gain defined as the ratio of the number of final electron-hole pairs to that created initially in the i-region assuming only electron injection at $x = W$, is given by [2]

$$M = \frac{1}{1 - \int_0^W \alpha(x) \exp\left(-\int_x^W (\alpha(x') - \beta(x'))\, dx'\right) dx} \tag{1}$$

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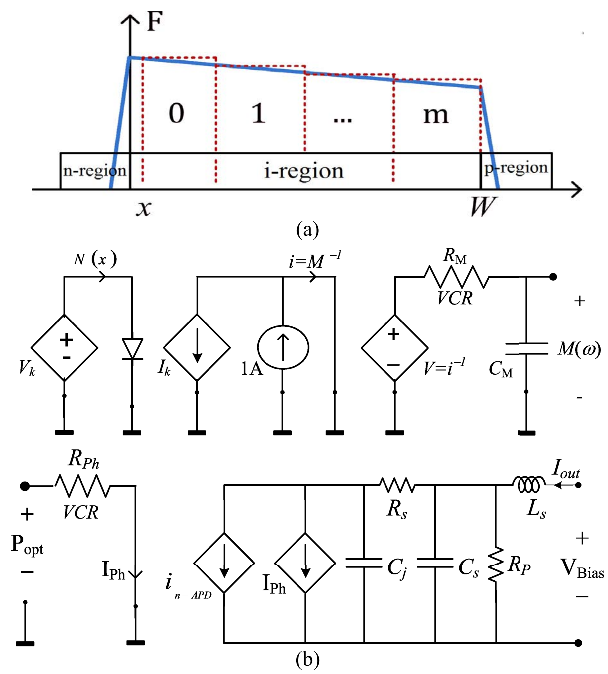

Fig. 1. (a) Electric field profile of a p-i-n avalanche photodiode. (b) An equivalent circuit model of noise for thin APDs.

where W represents the i-region thickness and $\alpha(x)$ and $\beta(x)$ are the electron and hole field dependent ionization coefficients respectively [4]. Since $\alpha(x)$ and $\beta(x)$ are dependent to carriers' position in the field, and in order to simplify (1) we divide the i-region from x to W into m segments with uniform local fields and define

$$\begin{aligned} N(x) &= \exp\left(-\int_x^W \left(\alpha(x') - \beta(x')\right) dx'\right) \\ &\approx \exp\Bigg(-\frac{W-x}{2m}\,[(\alpha(x) - \beta(x)) \\ &\qquad + 2\sum_{k=1}^{m-1}\left(\alpha\left(k\frac{W-x}{m}\right) - \beta\left(k\frac{W-x}{m}\right)\right) \\ &\qquad + (\alpha(W) - \beta(W))]\Bigg) \end{aligned} \tag{2}$$

As shown in Fig. 1(b), the argument of exponential function in (2) can be implemented as a series of $m+1$ weighted voltage sources, V_k. The summation of these voltages are then dropped over a diode to realize the exponential function and generate $N(x)$ as a current through it.

According to (2), for M we have

$$M^{-1} = 1-\int_0^W \alpha(x)N(x)\,dx \approx 1-\frac{W}{2m'}\left\{\alpha(0)N(0)+2\sum_{k=1}^{m-1}\alpha(kW/m')N\left(kW/m'\right) + \alpha(W)N(W)\right\} \quad (3)$$

where it is assumed that the i-region thickness is subdivided to m' segments. The terms in (3) are realized by m'+1 parallel current sources, I_k, and a DC current source in opposite direction as depicted in Fig. 1(b). The M at low frequencies is now achieved. Note that M and also $N(x)$ in (2) are in fact dimensionless and do not have the units of a current. However, to transform physical parameters to equivalent circuit parameters required for developing a circuit model, they are realized as a dependent current source. In order to consider the dependency of M on the input frequency variation [5]; $M(\omega) = M/(1+j\omega M\tau)$, the capacitor $C_M=\tau$ along with $R_M = M$ are included in the model. Since M is dependent on the electric field and thus bias voltage, R_M is in fact a voltage controlled resistor (VCR) available in SPICE-like simulators. In this equation, τ is the corrected intrinsic response time where for the sake of simplicity, it has been approximated with the multiplication region transit time [5].

The response of the APD as the multiplied photocurrent in response to an incident optical power can be obtained as [6]

$$I_{Ph} = q\eta\frac{P_{opt}}{h\nu}M \quad (4)$$

where P_{opt} is the incident optical power and η is the quantum efficiency defined as [6]

$$\eta = \eta_i\,(1-R)\,e^{-\alpha_p d}\left(1-\frac{e^{-\alpha_i W}}{1+\alpha_i L_p}\right) \quad (5)$$

where η_i is the internal quantum efficiency, R is the facet reflectivity, α_p and α_i are the absorption coefficient of the p- and i-region, respectively, d is the width of p-region and L_p is the diffusion length for holes. In Fig. 1(b), R_{Ph} with a value of $(h\nu)/(q\eta M)$ generates I_{Ph} in response to P_{opt}.

Assuming $\beta=k\alpha$, the excess noise factor $F(M)$ can be calculated as $F(M)=kM+(1-k)(2-M^{-1})$ and the power spectral density of the noise current in the APD is obtained as [6]

$$i^2_{n-APD} = 2qI_{Ph}\,|M(\omega)|^2\,F(M) \quad (6)$$

Note that for some materials like silicon, the approximation that $\beta/\alpha=k$, where k is a constant is not accurate. In such cases, it is recommended to use an effective k value. Based on (4) and (6), the output terminals of the device with I_{out} as the response current including noise effect are shown in Fig. 1(b) where R_S and L_S are the series parasitic resistance and inductance respectively, C_S and C_j are parasitic and junction capacitances respectively and R_P is the parallel parasitic resistance of the APD.

III. Discussion and Results

In order to obtain a more accurate circuit model for thin APDs we should consider the impacts of carrier's dead space as well as the history of carrier's previous ionization. These nonlocal effects are both dependent on the electric field and can be taken into account by defining an effective electric field for electrons and holes [1], [2] as a convenient approximation. According to one-dimensional Poisson's equation and using depletion approximation, the electric field profile in the lightly doped i-region can be written as

$$F(x) = qN_p x_p/\varepsilon_0\varepsilon_s - qN_i x/\varepsilon_0\varepsilon_s \quad (7)$$

where q is the electron charge, ε_0 and ε_s are the vacuum permittivity and the semiconductor relative permittivity, N_p and N_i are the doping concentration in the p- and i-region, respectively, and x_p is the depletion region width in the p- region obtained as given in [3]. To determine history dependent ionization coefficients and to treat the dead space in an analytical way, the effective electric field at position x for electrons and holes previously created at $x'=x-l_e$ and $x'=x+l_h$, respectively, either by impact ionization or optical generation is defined as

$$F_{eff,e,h}\left(x'|x\right) = \int_x^{x'=x\mp l_{e,h}} dx''F\left(x''\right)f_{ce,h}\left(x''|x\right) \quad (8)$$

where l_e and l_h are electron's and hole's dead space respectively, and $f_{ce,h}(x)$ (m^{-1}) is correlation function that is empirically found [1], [2] to provide good fit to experimental data and is defined as

$$f_{ce,h}\left(x'|x\right) = \frac{2}{\sqrt{\pi}\lambda_{e,h}}\exp\left(-\frac{\left(x'-x\right)^2}{\lambda^2_{e,h}}\right) \approx \frac{2}{\sqrt{\pi}\lambda_{e,h}}\exp\left(-\frac{l^2_{e,h}}{\lambda^2_{e,h}}\right) \quad (9)$$

In this equation, λ_e and λ_h are correlation length for electrons and holes respectively, obtained as [2]

$$\lambda_{e,h} \approx l_{e,h} = V_{de,h}\big/\overline{F} \quad (10)$$

where V_{de} and V_{dh} are voltage drops across the electron's and hole's dead space [2], respectively, and $\overline{F}$ is considered as the field in the beginning of each segment. By substituting (10) in (9) we reach to

$$f_{ce,h}\left(x'|x\right) = \frac{2}{\sqrt{\pi}l_{e,h}}\exp(-1) \approx 0.4\frac{F(x)}{V_{de,h}} \quad (11)$$

As a result, $F_{eff,e,h}(x)$ can be calculated easily over segments with width of l_e and l_h in the i-region. Using (8) the non-local history dependent ionization coefficients are represented by

$$\alpha,\beta(x'|x) = a_{n,p}\exp\left(\left(\frac{b_{n,p}}{F_{eff,e,h}(x'|x)}\right)^{c_{n,p}}\right) \quad (12)$$

where $a_{n,p}$, $b_{n,p}$, $c_{n,p}$ are constant and $\alpha(x'|x)dx$ and $\beta(x'|x)dx$ represent ionization probability at x for carriers generated at x' within the distance dx.

To verify the model capability for predicting the statistic response, a homojunction InAlAs PIN-APD is simulated for

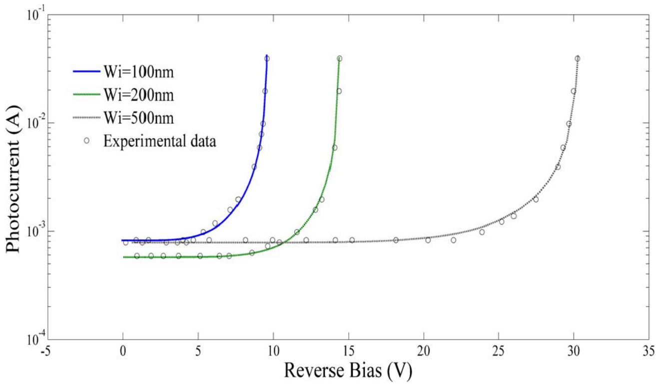

Fig. 2. Photocurrent versus reverse bias voltage for homojunction InAlAs APD with i-region thicknesses of $W = 100$, 200 and 500 nm (solid-, dashed-, and doted-lines) compared with experimental data provided by [7].

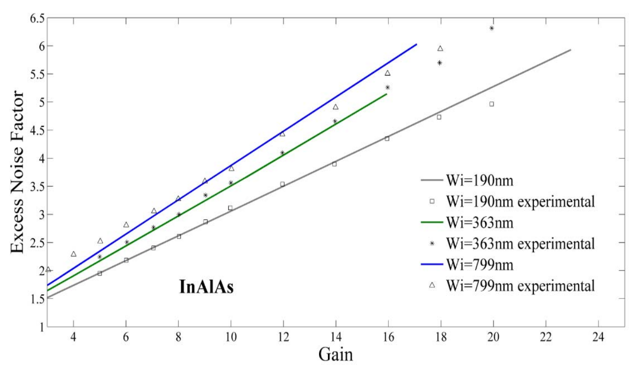

Fig. 3. Comparison of simulation and experimental results of excess noise factor for InAlAs APD [4] with different thicknesses of i-region; $W = 190$ nm, 363 nm and 799 nm.

different i-region thicknesses. As shown in Fig. 2, the results for photocurrent are in excellent agreement with experimental data provided by [7] as the model exhibits almost accurate estimation for current levels and breakdown voltages. Note that universal width-independent parameters given in [4] and the calculated effective electric field are utilized in the model to obtain ionization coefficients. Fig. 3 represents simulated excess noise factor of an InAlAs APD that well conforms to the experimental data provided in [4] for different thicknesses of the i-region.

The photocurrent results in Fig. 2 and the excess noise curves in Fig. 3 are all fitted using V_{de}=1.9 V and V_{dh}=2.5 V [1]. Using the non-local history dependent ionization coefficients also greatly helped the model to follow the experimental results especially when device thickness is reduced. In other words, if using local field theory where the effects of the dead space and carrier's history are not involved, the breakdown voltages will be estimated lower than actual values and the excess noise factor will be overestimated resulting to an inaccurate calculation of the noise current power spectral density. Finally, for a GaAs APD [8],

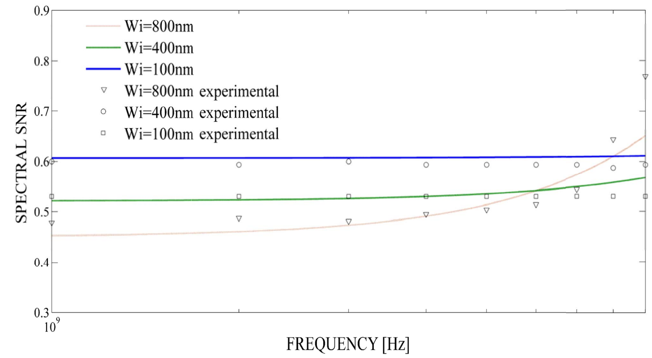

Fig. 4. Spectral SNR for GaAs APD with i-region thicknesses of $W = 100$, 400 and 800 nm (solid-, dashed- and doted-lines) compared with experimental data [8].

the spectral signal-to-noise ratio (SNR) defined as $SNR= I_{Ph}(\omega)/i_{n-APD}$ is calculated (Fig. 4). The increase in SNR beyond the bandwidth for 800 nm device is due to reduction in noise since multiplication phenomenon is vanishing [8]. The agreement between experimental results and those obtained from the model implies that, one can rely on this model for sensitivity analysis in link budget calculation.

IV. Conclusion

We have proposed a simple circuit model for thin APDs with the ability of multiplication noise calculation. In this model, nonlocal effects are treated by defining a position-dependent effective electric field profile. Time and frequency dependence of multiplication process is also included using a simple approach. This model is suitable for performance evaluation of optical systems at low and moderate data rates and can be simply extended to other types of APDs including SAGCM-APDs.

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