

Functional Process Capability Indices for Circular Profile

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Profile is a relation between one response variable and one or more explanatory variables that represent quality of a product or performance of a process. On the other hand, process capability indices are measures to help practitioners in improving the processes to satisfy the customer's expectations. Few researches are done to account for the process capability index in the areas of profile monitoring. All of these researches are focused on process capability index in simple linear profile. In all of these methods, response variables in different levels of explanatory variable are considered, and the relationship in all range of explanatory variable is neglected. In this paper, a functional method is proposed to measure process capability index of circular profiles in all range of explanatory variable. The proposed method follows the traditional definition of process capability indices. The functional method uses reference profile, functional specification limits and functional natural tolerance limits to present a functional form of process capability indices. This functional form results in measuring the process capability in each level of explanatory variable in circular profile as well as a unique value of process capability index for circular profile. The application of the proposed method is illustrated through a real case in automotive industry. Copyright © 2013 John Wiley & Sons, Ltd.

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1. Introduction

In statistical process control (SPC), we usually apply control charts to monitor quality of a process or product by using one or more quality characteristics. There are a noticeable number of situations in which the quality of process or product can be characterized by a relation (curve or profile) between one response variable and one or more explanatory variables rather than one or more quality characteristics in traditional SPC (Noorossana *et al.*¹). Profiles are commonly represented as parametric models such as simple linear regression, multiple linear regression, polynomial regression, nonlinear regression, logistic regression, circular models and cylindrical models. Various methods are presented for profile monitoring in both Phases I and II (Noorossana *et al.*¹). In Phase I, the goal is checking the stability of the process as well as parameters estimation while the aim in Phase II is detecting the shifts in the profile parameters as quickly as possible. Several researchers studied simple linear profile monitoring (see, e.g. Mestek *et al.*,² Stover and Brill,³ Kang and Albin,⁴ Kim *et al.*,⁵ Mahmoud *et al.*,⁶ Zou *et al.*,⁷ Croakin and Varner,⁸ Wang and Tsung,⁹ Gupta *et al.*¹⁰ and Saghaei *et al.*¹¹). Multiple linear and polynomial profiles are also studied by Zou *et al.*,¹² Mahmoud,¹³ Amiri *et al.*,¹⁴ Kazemzadeh *et al.*¹⁵ and Kazemzadeh *et al.*¹⁶ Other types of profile are also studied by many researchers. For a comprehensive review on profile monitoring, refer to Noorossana *et al.*¹

Roundness profile monitoring, which is recently considered by some researchers, well describe roundness that is a common geometric specification in industrial parts. Roundness profile has two types including circular and cylindrical profiles. Colosimo and Pacella¹⁷ used principle components analysis to identify systematic patterns in circular profiles. Colosimo *et al.*¹⁸ used a spatial autoregressive regression to model circular profile, and Pacella and Semeraro¹⁹ proposed an unsupervised neural network in monitoring circular profile.

On the other hand, process capability indices are used to evaluate the process performance (Bothe,²⁰ Pearn and Kotz²¹). Woodall²² suggested researches on assessing process capability with profile data. However, there are a few papers about the process capability index in profiles. Shahriari and Sarafian²³ presented a method for process capability index (C_{PK}) in circular profiles that is minimum value of process capability index of response variable in n levels of explanatory variable. Ebadi and Shahriari²⁴ used a multivariate process capability index for predicted response variable to present the process capability index in circular profile. Hoseinifard *et al.*²⁵ considered Gamma distribution for non-normal response variable and then used non-conforming proportion of response variable to analyze the process capability of simple linear profile. Hoseinifard and Abbasi²⁶ used non-conforming proportion of

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response variable to evaluate the process capability in simple linear profiles. Ebadi and Amiri²⁷ proposed three indices to measure the process capability in multivariate simple linear profile. In all of these methods, the process capability indices are defined on the response variable in n levels of explanatory variables.

Process capability in response variable or predicted response variable may ignore the relationship between the response variable and explanatory variables. In this paper, a functional method for measuring the process capability index of circular profile is proposed that utilizes all information in entire range of the explanatory variable.

The paper is organized as follows: In section 2, there is a brief background on the circular profile. In section 3, we propose a method to estimate the process capability indices in circular profiles. The case study is given in section 4, and, finally in section 5, we present the conclusions of the paper and some remarks for future researches.

2. Circular profile

In this section, a simple definition of roundness characteristics especially circular characteristic is given. Then, the model of circular profile is presented.

2.1. Roundness characteristics

Roundness is one of the most popular geometric characteristic in mechanical parts. This geometric characteristic is called circular and cylindrical profile in two-dimensional and three-dimensional spaces, respectively. The coordinate measuring machines (CMMs) are used for gathering data in roundness profile. This machine touches several points of circular characteristic of a round part in several angles and then determines the position of points in X and Y axis. An example of touched points is presented in Figure 1.

2.2. Circular characteristic modeling

The question arises after gathering the points is how circular characteristic can be modeled? Least square is the most popular method for fitting a circle to touched points.²³ In this method, a circle is fitted to the gathered points such that the sum square of distance of touched points from the fitted circle to be minimized (Figure 2).

If we have n touched points in n angles and R_i be the primal radius in i -th point respect to $(0,0)$, the radius of fitted circle (R) is calculated by Equation (1) as follows:

$$R = \frac{1}{n} \sum_{i=1}^n R_i \quad (1)$$

Assume that (a,b) is center of the fitted circle. The a and b are calculated by using Equations (2) and (3), respectively.

$$a = \frac{2}{n} \sum_{i=1}^n [R_i \cos(\theta_i)] \quad (2)$$

$$b = \frac{2}{n} \sum_{i=1}^n [R_i \sin(\theta_i)] \quad (3)$$

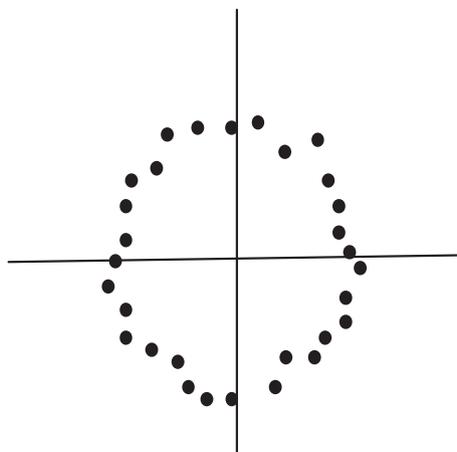


Figure 1. n touched points in a circular characteristic

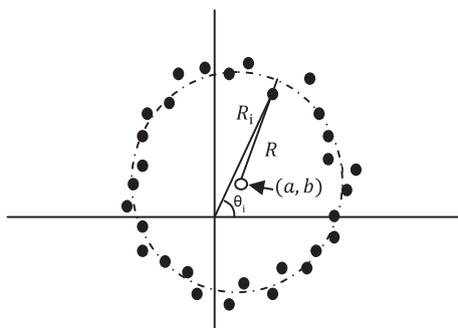


Figure 2. Schematic representation of least square circle

The error corresponding to the i -th touched point, which is difference between the real value of this point and the estimated value of the point based on fitted circle, is calculated by Equation (4).

$$\Delta_i = R_i - R - a\cos(\theta_i) - b\sin(\theta_i) \quad i = 1, 2, \dots, n \quad (4)$$

The estimated parameters of fitted circle in Equations (2) to (4) are obtained based on the least square method such that $\sum_{i=1}^n \Delta_i^2$ is minimized. The fitted circle is named least square circle (LSC). The general model of a circle by center of (a, b) is shown in Equation (5).

$$(x - a)^2 + (y - b)^2 = R^2 \quad (5)$$

2.3. Circular profile model

Polar and cartesian are two approaches to model the circular profiles. Figures 3 and 4 show the schematic representation of circle profiles in these two approaches.

Colosimo *et al.*¹⁸ proposed a cartesian parametric model of circular profile based on Fourier series which is represented in Equation (6).

$$y_j(i) = \sum_{k=2}^3 [b_{2i-1j} \cos(f_k(i-1)) + b_{2kj} \sin f_k(i-1)] + v_j(i), \quad (6)$$

where $j = 1, 2, \dots, m$, $i = 1, 2, \dots, n$. n is the number of points that are gathered for each circular profile, and m is the number of sample circular profiles. $y_j(i)$ is difference between the real circle radius and radius of LSC (R) in i -th point (R_i) in j -th circular profile. Also, $f_i = i(2\pi/\rho)$, $v_j(i)$ is corresponding error term, and b_{ij} 's are coefficients that should be estimated.

We consider a general polar model for circular profile as Equation (7).

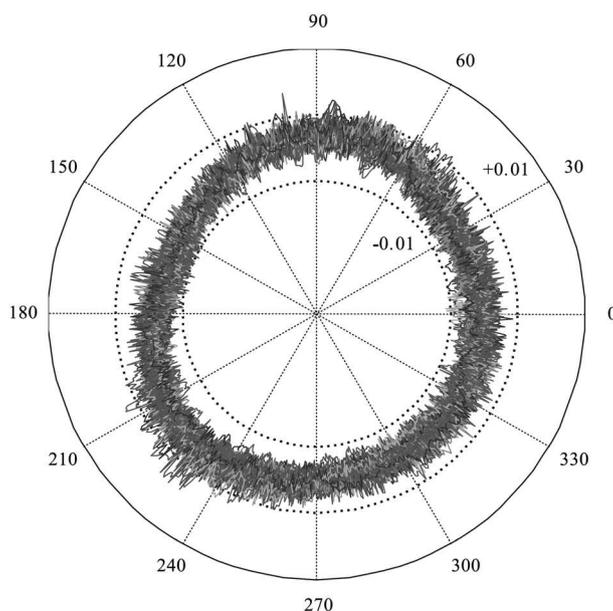


Figure 3. Polar representation of circular profile

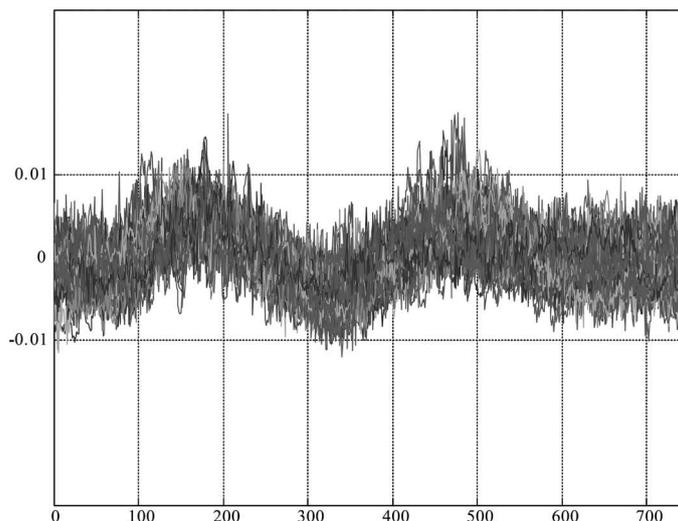


Figure 4. Cartesian representation of circular profile

$$R_{ij} = R_j + a_j \cos(\theta_i) + b_j \sin(\theta_i) + \Delta_{ij} \quad i = 1, 2, \dots, n; \quad j = 1, 2, \dots, k \quad (7)$$

where the pair observation (θ_i, R_{ij}) obtained in j^{th} random sample in which θ_i is the i^{th} design point for explanatory variable in j^{th} sample. Δ_{ij} s are values of independently normally distributed random variables with mean zero and variance equal to σ_j^2 .

In the next section, a method is proposed to calculate the process capability index of circular linear profile.

3. Functional process capability of circular profile

A new method is proposed to calculate the process capability index of circular linear profile. This method follows the concept of traditional definition of process capability index and proposes a functional form of process capability indices.

3.1. Traditional process capability indices

Process capability indices are widely used in assessing process performance. Kane²⁸ introduced the first process capability index as C_p . This index measures the potential capability of process with no attention to the process mean. C_p is presented in Equation (8).

$$C_p = \frac{USL - LSL}{UNTL - LNTL} = \frac{USL - LSL}{6\sigma} \quad (8)$$

where σ is the process standard deviation, LSL is lower specification limit and USL is upper specification limit. $UNTL$ is upper natural tolerance limit ($UNTL = \mu + 3\sigma$), and $LNTL$ is lower natural tolerance limit ($LNTL = \mu - 3\sigma$). C_{PK} is the mostly used process capability index because it compares process dispersion and tolerance range with considering the position of process mean.

$$C_{PK} = \min \left\{ \frac{USL - \mu}{UNTL - \mu}, \frac{\mu - LSL}{\mu - LNTL} \right\} = \min \left\{ \frac{USL - \mu}{3\sigma}, \frac{\mu - LSL}{3\sigma} \right\} \quad (9)$$

There are other types of process capability indices such as C_{PM} , C_{PMK} and S_{PK} for univariate situations (Kotz and Johnson²⁹).

Each of μ , LSL , USL , $LNTL$ and $UNTL$ is a point in traditional univariate process capability assessment, and we use differences of these points to evaluate the capability of process. However, in circular profile case, they are circles (Figures 5 and 6). Hence, differences between circles must be considered to determine process capability indices for circular profile.

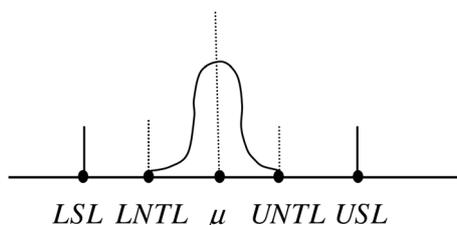


Figure 5. Tolerance and specification limits in traditional univariate quality characteristic

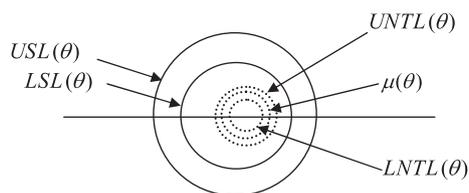


Figure 6. Tolerance and specification limits in circular profile

Natural tolerance and specification limits of circular profile have functional form. They get an especial value in each angle, and so there is an especial value for process capability Indices in each angle. Consequently, process capability indices of circular profile must have functional form based on angle. The functional form of process capability indices of circular profile are proposed in next section.

3.2. Proposed procedure

We propose a step-by-step procedure to evaluate process capability in circular profile.

Step1. Fit LSCs

The sample LSC is computed by using least square method in each profile. Equations (10) to (12) calculate radius and center of LSCs, respectively:

$$R_j = \frac{1}{n} \sum_{i=1}^n R_{ij} \quad j = 1, 2, \dots, k \quad (10)$$

$$a_j = \frac{2}{n} \sum_{i=1}^n [R_{ij} \cos(\theta_i)] \quad j = 1, 2, \dots, k \quad (11)$$

$$b_j = \frac{2}{n} \sum_{i=1}^n [R_{ij} \sin(\theta_i)] \quad j = 1, 2, \dots, k \quad (12)$$

Step2. Monitor circular profile in Phase I

We suggest a residual approach for monitoring circular profile in Phase I similar to the residual approach that Kang and Albine⁴ are proposed for simple linear profile monitoring in phase I. Assume there are k random samples of circular profile each of them contains n observations. For sample j , we have n residuals $\Delta_{ij}, i = 1, 2, \dots, n$ where Δ_{ij} is defined as $\Delta_{ij} = R_{ij} - R_j - a_j \cos(\theta_i) - b_j \sin(\theta_i)$. Define the average and the standard deviation of the residuals for sample j as $\bar{\Delta}_j = n^{-1} \sum_{i=1}^n \Delta_{ij}$ and S_{Δ_j} , respectively. We use the traditional $\bar{x} - S$ control charts to monitor the residuals. Suppose $\bar{\Delta}$ and \bar{S} are the mean of the averages and standard deviation of k random samples. Three sigma upper and lower control limits of related traditional \bar{x} control chart are $UCL = \bar{\Delta} + A_3 \bar{S}$ and $LCL = \bar{\Delta} - A_3 \bar{S}$, respectively. Three sigma upper and lower control limits of related traditional S control chart are $UCL = B_4 \bar{S}$ and $LCL = B_3 \bar{S}$, respectively.

Step3. Determine mean and natural tolerance limits

The reference (mean) circle can be computed as Equation (13).

$$R(\theta) = R + a \cos(\theta) + b \sin(\theta), \quad \theta \in [0, 2\pi] \quad (13)$$

where $R = (\sum_{j=1}^k R_j)/k$, $a = (\sum_{j=1}^k a_j)/k$, $b = (\sum_{j=1}^k b_j)/k$ and $R + a \cos(\theta) + b \sin(\theta)$ is a functional mean of response variable. The process capability index (C_p) defined in Equation (2) is a comparison between process natural tolerance limits and specification limits. $R(\theta)$ is a normal random variable with mean of $R + a \cos(\theta) + b \sin(\theta)$ and variance of σ^2 where $\sigma^2 = (\sum_{j=1}^k MSE_j)/k$. Hence, $UNTL$ and $LNTL$ can be defined as Equations (14) and (15), respectively.

$$UNTL_{R(\theta)} = R + a \cos(\theta) + b \sin(\theta) + 3\sigma, \quad (14)$$

$$LNTL_{R(\theta)} = R + a \cos(\theta) + b \sin(\theta) - 3\sigma. \quad (15)$$

It is obvious that $UNTL$ and $LNTL$ of $R(\theta)$ are two circles that have same centers at (a, b) and distance between their radius is equal to 6σ . As mentioned above, μ , $UNTL$ and $LNTL$ of $R(\theta)$ are functions of θ as shown in Equations (16) to (18).

$$\mu_{R(\theta)}(\theta) = R + a \cos(\theta) + b \sin(\theta), \tag{16}$$

$$UNTL_{R(\theta)}(\theta) = R + a \cos(\theta) + b \sin(\theta) + 3\sigma, \tag{17}$$

$$LNTL_{R(\theta)}(\theta) = R + a \cos(\theta) + b \sin(\theta) - 3\sigma. \tag{18}$$

Step4. Determine specification limits

Suppose r_u and r_l are the *USL* and *LSL* of radius, respectively, and consequently, the $USL_{R(\theta)}(\theta)$ and $LSL_{R(\theta)}(\theta)$ are as Equations (19) and (20), respectively.

$$USL_{R(\theta)}(\theta) = r_u, \tag{19}$$

$$LSL_{R(\theta)}(\theta) = r_l. \tag{20}$$

USL and *LSL* of $R(\theta)$ are two circles which have same centers at (0,0).

Step5. Determine C_p of circular profile

As mentioned above the $UNTL_{R(\theta)}(\theta)$, $LNTL_{R(\theta)}(\theta)$, $USL_{R(\theta)}(\theta)$ and $LSL_{R(\theta)}(\theta)$ are functional form of *UNTL*, *LNTL*, *USL* and *LSL*, respectively. By replacing functional forms of *UNTL*, *LNTL*, *USL* and *LSL* in traditional definition of C_p , a functional form of C_p as Equation (21) is computed.

$$C_p(\theta) = \frac{USL_{R(\theta)}(\theta) - LSL_{R(\theta)}(\theta)}{UNTL_{R(\theta)}(\theta) - LNTL_{R(\theta)}(\theta)} \quad \theta \in [0, 2\pi] \tag{21}$$

By using $C_p(\theta)$ as process capability index of circular profile, it is possible to evaluate the capability of process in each level of θ as explanatory variable. Process capability in each angle proposes detailed information of process. However, it is necessary to have a unique value of process capability index for circular profile in all range of angle to give an overall judgment about process capability. For this purpose, it is recommended to utilize area bounded between $USL_{R(\theta)}(\theta)$ and $LSL_{R(\theta)}(\theta)$ to compute $USL_{R(\theta)}(\theta) - LSL_{R(\theta)}(\theta)$ and also area bounded between $UNTL_{R(\theta)}(\theta)$ and $LNTL_{R(\theta)}(\theta)$ to compute $UNTL_{R(\theta)}(\theta) - LNTL_{R(\theta)}(\theta)$. Hence, Equation (22) is proposed to determine a unique value for C_p of circular profile.

$$C_p(\text{profile}) = \frac{\int_0^{2\pi} [USL_{R(\theta)}(\theta) - LSL_{R(\theta)}(\theta)] d\theta}{\int_0^{2\pi} [UNTL_{R(\theta)}(\theta) - LNTL_{R(\theta)}(\theta)] d\theta} \tag{22}$$

$USL_{R(\theta)}(\theta)$ and $LSL_{R(\theta)}(\theta)$ are co-centered circles at (0,0) and also $UNTL_{R(\theta)}(\theta)$ and $LNTL_{R(\theta)}(\theta)$ are co-centered circles at (a,b). Hence, $C_p(\text{profile})$ has a simple form as given in Equation (23).

$$C_p(\text{profile}) = \frac{\pi(r_u)^2 - \pi(r_l)^2}{\pi(R + 3\sigma)^2 - \pi(R - 3\sigma)^2} \tag{23}$$

Similar to the traditional univariate case, $C_p(\text{profile})$ shows the potential capability of process. $C_p(\text{profile})$ is greater than 1 if difference of $UNTL_{R(\theta)}(\theta)$, and $LNTL_{R(\theta)}(\theta)$ is less than difference of $USL_{R(\theta)}(\theta)$ and $LSL_{R(\theta)}(\theta)$. Figures 7 and 8 show that proposed functional C_p works

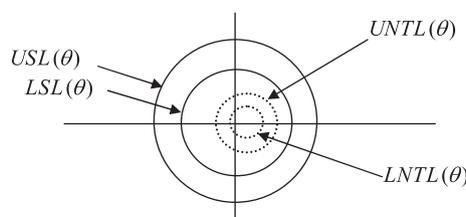


Figure 7. $C_p(\text{profile})$ is greater than 1

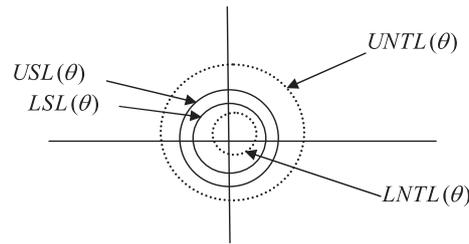


Figure 8. $C_p(\text{profile})$ is less than 1

similar to traditional C_p , $C_p(\text{profile})$ only show the potential capability of process. Figure 9 show an especial state that both process tolerance limits are greater than both specification limits but $C_p(\text{profile})$ is greater than 1. Consequently, we must determine C_{PK} of circular profile to get actual capability of process.

Step6. **Determine C_{PK} of circular profile**

Similarly, functional C_{PK} of circular profile is presented as Equation (24).

$$C_{PK}(\theta) = \min \left\{ \frac{USL_{R(\theta)}(\theta) - \mu_{R(\theta)}(\theta)}{UNTL_{R(\theta)}(\theta) - \mu_{R(\theta)}(\theta)}, \frac{\mu_{R(\theta)}(\theta) - LSL_{R(\theta)}(\theta)}{\mu_{R(\theta)}(\theta) - LNTL_{R(\theta)}(\theta)} \right\} \quad \theta \in [0, 2\pi] \quad (24)$$

$C_{PK}(\theta)$ gives the value of C_{PK} of circular profile in each level of θ . Equation (22) can be used to compute a unique value for C_{PK} of circular profile.

$$C_{PK}(\text{profile}) = \min \left\{ \frac{\int_0^{2\pi} [USL_{R(\theta)}(\theta) - \mu_{R(\theta)}(\theta)] d\theta}{\int_0^{2\pi} [UNTL_{R(\theta)}(\theta) - \mu_{R(\theta)}(\theta)] d\theta}, \frac{\int_0^{2\pi} [\mu_{R(\theta)}(\theta) - LSL_{R(\theta)}(\theta)] d\theta}{\int_0^{2\pi} [\mu_{R(\theta)}(\theta) - LNTL_{R(\theta)}(\theta)] d\theta} \right\} \quad (24)$$

Three examples of relationship between $C_p(\text{profile})$ and $C_{PK}(\text{profile})$ are represented in Figures 10, 11 and 12 which are similar to the relationship between C_p and C_{PK} in traditional process capability analysis.

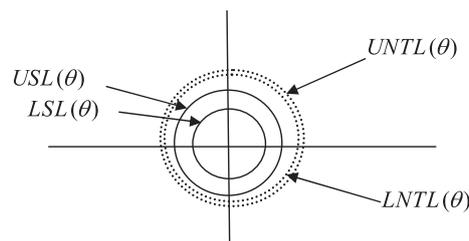


Figure 9. $C_p(\text{profile})$ is greater than 1

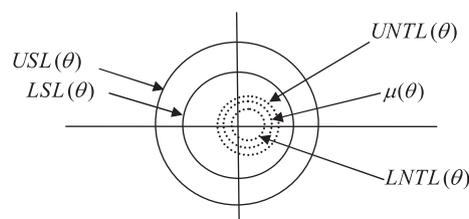


Figure 10. $C_p(\text{profile})$ is greater than 1 but $C_{PK}(\text{profile})$ is less than 0

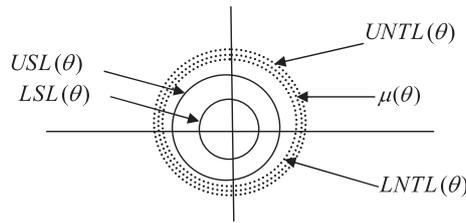


Figure 11. $C_p(\text{profile})$ is greater than 1 but $C_{pk}(\text{profile})$ is less than 0

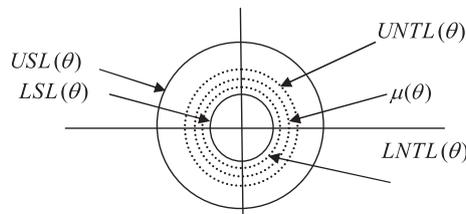


Figure 12. $C_p(\text{profile})$ is greater than 1 and $C_{pk}(\text{profile})$ is greater than 1

The process capability index (C_{PK}) when only upper or lower functional specification limit is available can be computed as Equations (25) and (26), respectively.

$$C_{PU}(\text{profile}) = \frac{\int_0^{2\pi} [USL_{R(\theta)}(\theta) - \mu_{R(\theta)}(\theta)] d\theta}{\int_0^{2\pi} [UNTL_{R(\theta)}(\theta) - \mu_{R(\theta)}(\theta)] d\theta}, \quad (25)$$

$$C_{PL}(\text{profile}) = \frac{\int_0^{2\pi} [\mu_{R(\theta)}(\theta) - LSL_{R(\theta)}(\theta)] d\theta}{\int_0^{2\pi} [\mu_{R(\theta)}(\theta) - LNTL_{R(\theta)}(\theta)] d\theta}. \quad (26)$$

When $\mu_{R(\theta)}(\theta)$, $UNTL_{R(\theta)}(\theta)$, $LNTL_{R(\theta)}(\theta)$, $USL_{R(\theta)}(\theta)$ and $LSL_{R(\theta)}(\theta)$ are co-centered circles at (0,0), $C_{PK}(\text{profile})$ has a simple form as Equation (27).

$$C_{PK}(\text{profile}) = \min \left\{ \frac{\pi r_u^2 - \pi R^2}{\pi(R + 3\sigma)^2 - \pi R^2}, \frac{\pi R^2 - \pi r_l^2}{\pi R^2 - \pi(R - 3\sigma)^2} \right\} \quad (27)$$

It is obvious that $\mu_{R(\theta)}(\theta)$ is greater than $LNTL_{R(\theta)}(\theta)$ and less than $UNTL_{R(\theta)}(\theta)$ in each case of circular profile. However, there is not a predefined relationship between $\mu_{R(\theta)}(\theta)$ and $USL_{R(\theta)}(\theta)$ or $LSL_{R(\theta)}(\theta)$. Hence, it is essential to use proper signed bounded area between $\mu_{R(\theta)}(\theta)$ and $USL_{R(\theta)}(\theta)$ to compute $USL_{R(\theta)}(\theta) - \mu_{R(\theta)}(\theta)$ in $C_{PK}(\text{profile})$ and $C_{PU}(\text{profile})$ and also $\mu_{R(\theta)}(\theta) - LSL_{R(\theta)}(\theta)$ in $C_{PK}(\text{profile})$ and $C_{PL}(\text{profile})$.

Assume for example, $USL_{R(\theta)}(\theta)$ is less than $\mu_{R(\theta)}(\theta)$ in $[0, \pi]$ and greater than $\mu_{R(\theta)}(\theta)$ in remained range of $[\pi, 2\pi]$. Thus, The formula of C_{PU} must be changed as Equation (28).

$$C_{PU}(\text{profile}) = \frac{-\int_0^{\pi} [USL_{R(\theta)}(\theta) - \mu_{R(\theta)}(\theta)] d\theta + \int_{\pi}^{2\pi} [USL_{R(\theta)}(\theta) - \mu_{R(\theta)}(\theta)] d\theta}{\int_0^{2\pi} [UNTL_{R(\theta)}(\theta) - \mu_{R(\theta)}(\theta)] d\theta} \quad (28)$$

In the next section, the application of the proposed method is shown in a case study in automobile part-making industry.

4. Case study

Wheel Plate (Figure 13) is a round automobile part that a circular profile is fitted to its middle section.

Statistical quality control policy of factory about this circular quality characteristic is that one produced part is sampled every week and touched in eight points by CMM in laboratory and then the radius of LSC is calculated for each product. Designers propose

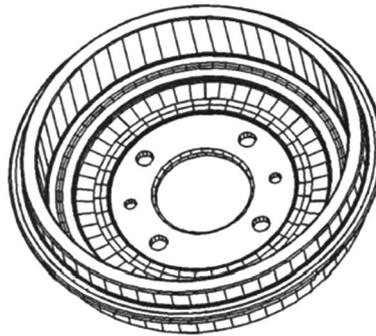


Figure 13. Wheel plate

specification limits of radius as $LSL = 114.21$ and $USL = 114.39$. There is no insight about performance of the process in this quality characteristic. The eight touched points of 49 produced parts are gathered as random samples. The related circular profile can be modeled as below:

$$R_{ij} = R_j + a_j \cos(\theta_i) + b_j \sin(\theta_i) + \Delta_{ij} \quad i = 1, 2, \dots, 8; \quad j = 1, 2, \dots, 49$$

Data set of estimated R , a , b and σ^2 for 49 LSCs are shown in Table I.

Figure 14 shows the control charts for monitoring circular profile in Phase I. The samples 11 up to 43 are omitted, and only 16 samples are remained.

The average of estimated parameters in 16 remained circular profiles leads to $R = 114.331$, $a = -0.0004$, $b = -0.0025$ and $\sigma^2 = 0.000069$.

As mentioned above, a and b are very close to 0. Figure 15 shows the pattern of primal 49 circular profiles. It is obvious that distances of a and b to 0 with considering the scale and values of radius are neglectable. Hence, suppose that $(a, b) = (0, 0)$ and hence $LSL(\theta)$, $USL(\theta)$, $\mu(\theta)$, $UNTL(\theta)$ and $LNTL(\theta)$ are same in center that is $(0, 0)$. Characteristics of these limits are shown in Table II.

The pattern of limits is shown in Figure 16 which is similar to Figure 11, and it is clear that both $C_p(\text{profile})$ and $C_{pk}(\text{profile})$ should be greater than 1.

Table I. Dataset of sample least square circles

Part	R	a	b	σ^2	Part	R	a	b	σ^2
1	114.315	-0.019	0.005	0.00012	26	114.35	-0.014	0.005	4.6E-05
2	114.3675	0.006	-0.001	9.3E-05	27	114.337	-0.002	-0.013	4.4E-05
3	114.3625	0.004	0.003	0.00012	28	114.3305	-0.002	-0.008	0.00016
4	114.3245	0.012	0.005	9.5E-05	29	114.3295	0.002	0.000	8.9E-05
5	114.3455	0.001	-0.008	5.6E-05	30	114.3155	-0.007	-0.001	0.00019
6	114.329	-0.019	0.001	7.8E-05	31	114.353	-0.024	0.011	2.5E-05
7	114.318	0.002	-0.015	3.9E-05	32	114.3465	-0.006	0.007	7.5E-05
8	114.351	-0.004	-0.002	9.4E-06	33	114.356	-0.007	-0.008	3.8E-05
9	114.337	0.006	0.013	6.2E-05	34	114.349	-0.002	-0.014	0.00005
10	114.3165	-0.005	-0.001	1.5E-05	35	114.328	-0.001	-0.007	0.00015
11	114.34	-0.008	-0.001	5.2E-05	36	114.317	-0.003	0.004	0.00035
12	114.351	0.003	-0.009	7.7E-05	37	114.3425	-0.009	-0.008	0.00026
13	114.349	-0.001	0.001	7.7E-06	38	114.34	-0.005	0.004	0.00017
14	114.351	-0.006	-0.012	9E-05	39	114.3365	0.003	-0.002	0.00038
15	114.338	0.001	-0.002	0.0001	40	114.3425	-0.016	-0.014	0.00018
16	114.35	-0.500	-0.008	5.7E-05	41	114.331	-0.010	0.001	7E-05
17	114.327	-0.001	-0.009	4.3E-05	42	114.3215	0.006	0.012	6E-05
18	114.3425	0.006	0.005	3.2E-05	43	114.3325	0.006	0.005	7.9E-05
19	114.3605	-0.009	-0.015	3.6E-05	44	114.337	0.008	0.013	4E-05
20	114.3255	-0.004	0.005	9E-05	45	114.3195	0.010	-0.003	3.1E-05
21	114.339	0.005	-0.014	4.5E-05	46	114.3195	-0.008	-0.020	0.00014
22	114.333	-0.033	-0.003	7.1E-05	47	114.3135	-0.006	-0.010	4.9E-05
23	114.3165	0.011	0.008	0.00014	48	114.3195	0.006	-0.009	7.3E-05
24	114.349	-0.014	-0.014	0.00012	49	114.3195	0.000	-0.011	8.6E-05
25	114.325	0.012	0.007	9E-05					

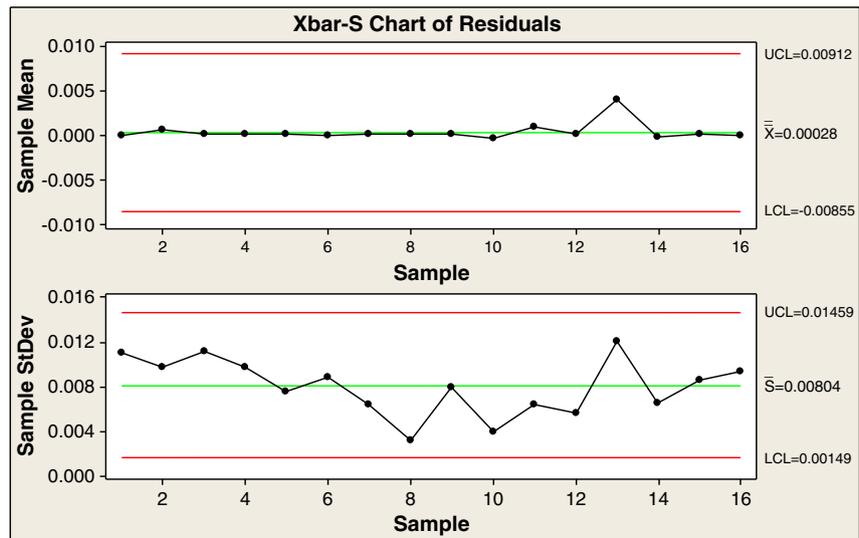


Figure 14. $\bar{x} - s$ control charts of residuals in Phase I

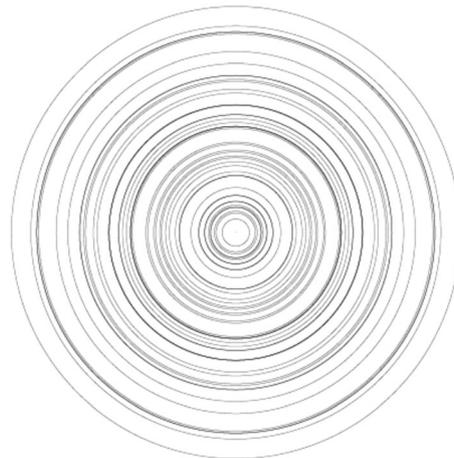


Figure 15. Pattern of 49 primal least square circles

Table II. Characteristics of limits		
Limit	Center	Radius
$LSL(\theta)$	(0,0)	114.21
$USL(\theta)$	(0,0)	114.39
$\mu(\theta)$	(0,0)	114.337
$UNTL(\theta)$	(0,0)	114.356
$LNTL(\theta)$	(0,0)	114.306

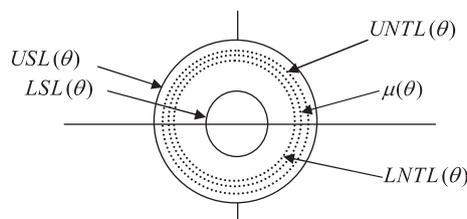


Figure 16. Pattern of limits in case study

Equation (20) gives $C_p(\text{profile})$ as below,

$$C_p(\text{profile}) = \frac{\pi(114.39)^2 - \pi(114.21)^2}{\pi(114.331 + 3 \times 0.00833)^2 - \pi(114.331 - 3 \times 0.00833)^2} = 3.60$$

And Equation (25) gives $C_{PK}(\text{profile})$ as below:

$$C_{PK}(\text{profile}) = \min \left\{ \frac{\pi(114.39)^2 - \pi(114.331)^2}{\pi(114.331 + 3 \times 0.00833)^2 - \pi(11.331)^2}, \frac{\pi(11.331)^2 - \pi(11.21)^2}{\pi(11.331)^2 - \pi(11.331 - 3 \times 0.00833)^2} \right\} = 2.36$$

We can conclude that process has a high capability in circular characteristic generation in wheel plate.

5. Conclusion

In this paper, a new method to determine a process capability index for circular profile was proposed. In this new method, process capability index defined as a functional form of the angle as explanatory variable. This index computes the capability of the process in each level of explanatory variable. In addition, the area bounded between mean, specification limits and natural tolerance limits was used to compute a unique value for circular profile. By using this concept, the traditional C_p and C_{PK} were generalized to the process capability indices for circular profile. The application of the proposed method was illustrated through a case on an automobile part, wheel plate. The new approach proposed in this paper can be easily used for various industrial round parts. Investigating the applicability of the proposed approach in other types of profiles such as simple linear profiles, polynomial profiles and nonlinear profiles can be a fruitful area of research.

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