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Multi-objective Economic-Statistical Design of Cumulative Count of Conforming Control Chart

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ABSTRACT

Cumulative Count of Conforming (CCC) charts are utilized for monitoring the quality characteristics in high-quality processes. Executive cost of control charts is a motivation for researchers to design them with the lowest cost. Usually in most researches, only one objective named cost function is minimized subject to statistical constraints, which is not effective method for economic-statistical design of control charts. In this paper, a multi-objective model for the economic-statistical design of the CCC control chart is developed. Then, multi-objective evolutionary algorithm (NSGA-II) for obtaining the Pareto optimal solution of the model is proposed. A numerical example is applied to illustrate the effectiveness of the proposed model. This model leads to lower cost and smaller probability of Type I and Type II errors, compared with economic model. In addition, a sensitivity analysis is done to investigate the effect of input parameters on the best solutions of the proposed model.

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1. INTRODUCTION

Control chart as one of Statistical Process Control (SPC) tools, has a very significant role in achieving process stability. However, traditional control charts face a number of problems in high-quality processes, which are very common in the modern manufacturing environments. Cumulative Count of Conforming (CCC) control charts is useful procedure for process control when large number of consecutive conforming items are perceived between two nonconforming ones. The CCC chart is useful for one-at-time inspections which are common in automated manufacturing processes. Generally, it is a technique for high-quality processes when nonconforming items are observed infrequently. This chart is statistically based on the geometric distribution. Firstly, Calvin [1] was developed the idea of tracking cumulative counts to monitor assignable causes in high quality process. Then introduction of CCC charts is presented by Goh [2] The use of CCC control charts has been further studied by Lucas [3], Glushkovsky [4] and Xie and Goh [5]. Chan et al. [6] use the idea of counting the cumulative conforming items in the case of continuous production process. Similar to other control charts, an important stage in CCC chart implementation is its design. A common practice is to design the control chart with primarily statistical considerations.

The design of a control chart includes determining the sample size (n), the sampling frequency or time interval between samples (h), and the coefficient of control limits (l). One of the approaches for design of a control chart is statistical design in which only statistical properties such as probability of type I and type II errors are considered. However, design of control chart includes various expenses, such as the costs of sampling and checking, costs associated with examining out-of-control signals, the cost of locating and repairing assignable causes and costs of allowing nonconforming products to reach the customer. These expenses are not considered in statistical design. Hence, it is very rational to consider the design of control chart from an economic perspective. Economic design of control charts is another approach, in which a

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cost model for particular type of industrial process is developed and optimization methodologies search the optimal solutions that minimize the expected cost per hour.

In the literature, many models have been developed for economic design of control charts. First time, Weiler [7] attempted to design a control chart based on an economic criterion. Executive cost of control chart cause that researchers design control charts economically based on either minimizing the cost or maximizing the benefit. Duncan [8] presented an economic model for $\bar{X}$ control chart that this model can be used in the most types of control charts. Also, Lorenzen and Vance [9] proposed another cost function for economic design of control charts. This model permits production to be continued or stopped during search or repair. These are two famous cost models used by many researchers in economic design of many control charts. McWilliams [10] and Surtihadi and Rachavachari [11] presented economic design of $\bar{X}$ chart with Lorenzen and Vance (LV) cost function. Also, Montgomery et al. [12] used the LV cost function to economic design of EWMA chart and Simpson and Keats [13] applied it to economic model of CUSUM chart. Collani et al. [14] considered economic design of control charts for monitoring the nonconforming probability. Xie et al. [15] presented economic design of cumulative count of conforming (CCC) chart. They used LV cost function for economic model of CCC chart. Zhang et al. [16] extended economic design of cumulative count of conforming charts under inspection by samples. They proposed this model to monitor the cumulative number of samples inspected until a nonconforming sample is encountered.

Woodall [17] stated the economic design of control charts leads to poor statistical properties. So, economic-statistical design of control chart has been proposed to solve this problem. Saniga [18] was the first one who presented the economic-statistical design of $\bar{X}$ and R control charts including the economic objective and statistical constraints.

Multi-objective economic-statistical design of control charts is a new approach proposed by some researchers to design a control chart from both points of economic and statistical properties. Firstly, Evans and Emberton [19] introduced multi-objective economic-statistical design for joint $\bar{X}$ and R control charts. They considered cost function and statistical properties as objectives that are optimized simultaneously. Hence, optimal design of a control chart can be considered as a multiple criteria decision-making (MCDM) problem. Del Castillo et al. [20] proposed a multi-objective approach in designing joint $\bar{X}$ and R control charts by using MCDM approach. Also, Celano and Fishera [21] presented a multi-objective model based on Duncan [8] cost model for design of $\bar{X}$ control chart and they optimized this model using genetic algorithm (GA). Zarandi et al. [22] used fuzzy cost parameters for multi-objective design of $\bar{X}$ control chart. They used adaptive neuro-fuzzy interface system (ANFIS) and GA to obtain optimal parameters of the $\bar{X}$ control chart. In addition, Chen and Liao [23] and Asadzadeh and Khoshalhan [24] used multi-objective approach for designing $\bar{X}$ control chart. Moreover, Amiri et al. [25] presented multi-objective economic-statistical model of MEWMA control charts and applied GA algorithm for obtaining optimal solutions. Amiri et al. [26] also considered multi-objective economic-statistical design of EWMA control chart. Bashiri et al. [27, 28] considered multi-objective economic-statistical design of $\bar{X}$ and np control charts, respectively. Safaei et al. [29] also investigated multi-objective economic-statistical design of $\bar{X}$ control chart.

Review of the literature shows that multi-objective economic-statistical design of the CCC control chart has not been yet considered by the researchers. Therefore, in this paper, we extend a multi-objective economic-statistical design of the CCC control chart by Lorenzen and Vance cost function. The design parameters of the CCC control chart are obtained such that the expected cost per hour as well as out-of-control average run length (ARL_1) are minimized, while desired in-control average run length (ARL_0) is obtained. In addition, a non-dominated sorting genetic algorithm II (NSGA-II) is applied to solve the proposed multi-objective model. Because, the objectives are nonlinear and complex and the NSGA-II algorithm has some advantages and leads to accpetable solutions in short time. The advantages of the NSGAII algorithm will be discussed in section 4.

The remainder of this paper is organized as follows: In section 2, first the CCC control chart is introduced briefly then the Lorenzen and Vance cost function used in economic design of the CCC control chart is discussed. In section 3, multi-objective model for economic-statistical design of CCC control charts is developed. In section 4, the NSGA-II algorithm as the optimization method is proposed for the multi-objective economic-statistical design of control charts. In section 5, a numerical example is presented and the effectiveness of the multi-objective model is illustrated. Also, this proposed model is compared with economic model. In section 6, sensitivity analysis on the parameters of the proposed model is presented. Finally, the conclusion is mentioned in section 7.

2. THE ECONOMIC MODEL FOR CCC CHART

2. 1. CCC Control Charts The idea of the CCC

control chart was first developed by Calvin [1]. Suppose x is the number of observed items before a nonconforming item. Then, x is a geometric random variable with probability function:

$$p(1-p)^{x-1} \quad x = 1,2,\ldots, \tag{1}$$

where, p is probability of observed nonconforming item.

Average of geometric distribution is considered as center line of the CCC control chart:

$$CL = 1/p \tag{2}$$

Suppose false alarm probability equals to α. Control limits for the CCC control chart based on the geometric distribution are derived from the cumulative distribution function of x. These control limits are as follows:

$$LCL = \frac{ln(\alpha/2)}{ln(1-p)} \tag{3}$$

$$UCL = \frac{ln(1-\alpha/2)}{ln(1-p)} \tag{4}$$

Design parameters of the CCC control charts are including the coefficient of control limit for the CCC control chart (l) and sampling interval or time between two consecutive samples (h). In CCC control charts, sample size (n) is equal to 1 because sampling procedure is based on one-at-a-time. The machanism of the CCC control charts by using a flowchart is shown in Figure 1.

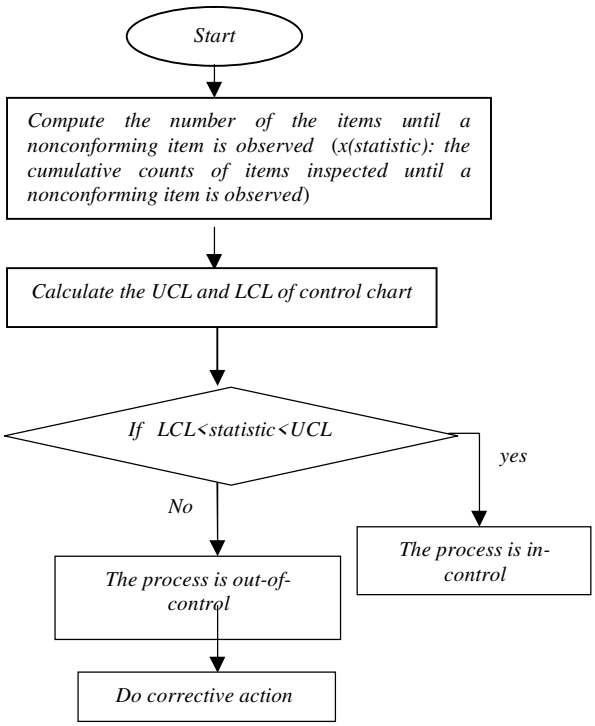

Figure 1. Machanism of the CCC control chart

2. 2. The Cost Function In this paper, Lorenzen and Vance [9] cost function is used in the formulation of the cost function for the CCC control chart. In this cost function, minimizing the expected cost in unit of time that is computed by dividing the expected total cost in a cycle by the expected cycle time is used. The total cost in a cycle involves sampling inspection, search and repair costs in addition to the cost due to producing nonconforming items. The parameters of this cost model are notated as follows:

Cost and time parameters

C_0	quality cost per hour due to nonconformities produced while production process is in-control
C_1	quality cost per hour due to nonconformities produced while production process is out-of-control
a	fixed cost per sampling
b	cost per unit sampling
Y	cost per false alarm
W	cost to located and repair the assignable cause
E	time to sampling and chart one item
T_0	expected search time when signal is a false alarm
T_1	expected time to detect the assignable cause
T_2	expected time to repair the process

Process parameters

p_0	expected nonconforming productions when process is in-control
p_1	expected nonconforming productions when process is out-of-control

$$\delta_1 = \begin{cases} 1 & \text{if production process continues during search} \\ 0 & \text{if production process stops during search} \end{cases}$$

$$\delta_2 = \begin{cases} 1 & \text{if production process continues during repair} \\ 0 & \text{if production process stops during repair} \end{cases}$$

Dependent parameters

ARL_0	average run length when process is in-control
ARL_1	average run length when process is out-of-control
s	expected number of samples taken when process is in-control
τ	expected time of occurrence of assignable cause

The process is assumed to begin in a state of statistical control with a known fraction non-conforming p_0. Assignable cause occurrence causes the fraction nonconforming of process changes to p_1. The time between events of the assignable cause is exponentially distributed with a mean λ.

The expected cost per hour for economic model of CCC control chart is defined as follows:

$$C = C_0 + \frac{S_1}{D} + \frac{a+b}{h} \times \frac{S_2}{D} \quad , \tag{5}$$

where S_1, S_2 and D are defined in Equations (6) to (8), respectively.

$$S_1 = \lambda(C_1 - C_0)(-\tau + E + h(ARL_1) + T_1 + T_2) - \lambda C_1[(1-\delta_1)T_1 + (1-\delta_2)T_2] + \frac{\lambda s Y}{ARL_0} - \frac{\lambda C_0(1-\delta_1)sT_0}{ARL_0} + \lambda W \quad (6)$$

$$S_2 = 1 + \lambda(-\tau + E + h(ARL_1) + \delta_1 T_1 + \delta_2 T_2) \quad (7)$$

and

$$D = 1 + \frac{(1-\delta_1)sT_0}{ARL_0} + \lambda(-\tau + E + h(ARL_1) + T_1 + T_2) \quad (8)$$

So, expected number of samples when process is in-control (s) and expected time of occurrence of assignable cause (τ) are obtained as follows:

$$s = \frac{1}{e^{\lambda h} - 1} \quad (9)$$

and

$$\tau = \frac{1}{\lambda} - \frac{h}{e^{\lambda h} - 1} \quad (10)$$

In this paper, only lower control limit for the CCC control chart is investigated due to its practicality and simplicity. When the CCC statistic falls out of upper control limit, the fraction of nonconforming in the process is decreased. Hence, we ignore the UCL of the CCC control chart and use a one-sided CCC control chart. The number of consecutive conforming items is counted up to the time which a nonconforming one is detected. If the count is smaller than or equal to lower control limit (LCL), assignable causes is taken place.

3. MULTI-OBJECTIVE ECONOMIC-STATISTICAL MODEL

The main issue with the economic design of control chart is ignoring statistical properties. Therefore, to overcome this problem economic design of control chart is completed by statistical constraints and economic-statistical design is formed. Usually, in economic-statistical design of control charts, cost function is minimized as single objective subjected to statistical properties. Statistical properties including probability of Type I and II error or in-control average run length ($\boldsymbol{ARL_0}$) and out-of-control average run length ($\boldsymbol{ARL_1}$). In this paper, we apply statistical properties of $\boldsymbol{ARL_0}$ and $\boldsymbol{ARL_1}$. Economic-statistical design of control charts leads to better results in both economic and statistical properties.

In this paper, we develop multi-objective economic-statistical design of Cumulative Count of Conforming control charts (CCC). In this multi-objective model, optimal values of sampling interval (h) and the coefficient of the lower control limit (l) are obtained such that cost function as well as out-of-control average run length (ARL_1) are minimized, while in-control average run length (ARL_0) is a statistical constraint.

The proposed multi-objective model for CCC control charts is as follows:

$$\begin{aligned} &min\, C(h,l) \\ &min\, ARL_1 \\ &\textit{Subject to :} \\ &\quad ARL_0 > ARL_L \\ &\quad h \textit{ and } l > 0\ , \end{aligned} \quad (11)$$

where $C(h,l)$ is economic cost function of CCC control chart presented in Equation (1). The ARL_L is the lower bound for in-control ARL_0 Also, in-control average run length (ARL_0) and out-of-control average run length (ARL_1) for CCC control charts are obtained by Equations (12) and (13).

$$ARL_0 = l + \frac{1-p_0}{p_0} + \frac{1}{p_0[1-(1-p_0)^{l-1}]} \quad (12)$$

$$ARL_1 = l + \frac{1-p_1}{p_1} + \frac{1}{p_1[1-(1-p_1)^{l-1}]} \quad (13)$$

To decrease the total cost, the average run length should be large when the process is in-control (ARL_0) and it should be small when the process is out-of-control (ARL_1).

4. OPTIMIZATION ALGORITHM

To obtain the optimal solution of the multi-objective economic-statistical models, different algorithms are developed. The quality of a Pareto optimal solution by three desirable properties, involving diversity, uniformity (a uniform distribution of non-dominated solutions) and cardinality can be appraised. Several researchers presented various evolutionary algorithms to obtain the Pareto optimal solutions with the above mentioned properties. For example, Fonseca and Fleming [30] proposed multi-objective genetic algorithm or Deb [31] suggested multi-objective Tabu search and non-dominated sorting genetic algorithm (NSGA and NSGA-II). NSGA-II algorithm is an effective method to recognize the Pareto optimal set. This algorithm is used to search for non-dominated solutions. Among the benefits of NSGA-II algorithm is utilizing the two ranking and crowding distance functions to obtain better optimum solutions. In the ranking function (non-domination sorting), the solutions are ranked by using the non-domination criteria and the solutions that have the similar ranks are placed in a front. In the crowding distance function, the crowding distance between both solutions in a front is calculated. The aim of this function is setting the

solutions in the overall front. In this evolutionary algorithm (NSGA-II), the solution with the lowest rank and the most crowding distance is the optimum solution. Finally, the NSGA-II algorithm leads to more accurate and better solutions.

In this section, we apply NSGA-II algorithm to solve the multi-objective economic-statistical model of CCC control charts and to obtain Pareto optimal set. The decision variables of the proposed model are sampling interval (h) and the coefficient of lower control limit (l). Figure 2 illustrates a pattern of a chromosome. Hence, a chromosome consists of two genes, as decision variable. The chromosomes are compared based on their fitness function.

Steps of the NSGA-II algorithm for finding a Pareto optimal set of multi-objective model are defined as follows:

1. Generate initial population of size n-pop (the number of chromosomes), randomly.
2. Calculate out-of-control average run length (ARL_1), in-control average run length (ARL_0) and cost function (C) for each chromosome.
3. By using non-domination criteria, rank the initial population
4. Compute crowding distance for the initial population.
5. Use the crossover and mutation operator to generate intermediate population of size n-pop
6. Evaluate objectives (C and ARL_1) and constraint (ARL_0) for this created intermediate population.
7. Combine the parent and intermediate populations, then rank them and compute the crowding distance.
8. Select population that has best individuals based on the rank and crowding distance criteria as a new population of size n-pop.
9. Go to step 3 and repeat the steps until the stopping rule (number of generations) occurs.

In this algorithm, crossover operator is used with the probability of 0.8. The mutation operator creates the mutated children using adaptive mutation of the genes with the probability of 0.3. Also, the crowding distance means the relative closeness of a solution to other solutions in the population and is applied for the solutions in the same rank or in each front. Eventually, chromosomes with Pareto optimal values are reported as the Pareto solution set for multi-objective economic-statistical design of the CCC control chart.

5. NUMERICAL EXAMPLE

In this section, we evaluate the performance of the proposed method using a numerical example. Lorenzen and Vance [9] depicted a foundry operation which produces 84 casting per hour. In castings production, a standard is set to proscribe high carbon-silicate content as it will result in low tensile strength. In this place, we consider the case where castings production process is a high quality process with very low fraction nonconforming. Therefore, the CCC control chart should be used because nonconforming items occur rarely in process. Suppose in-control fraction nonconforming items $p_0 = 0.0001$ and out-of-control fractions nonconforming items, p_1 varying from 0.0005 to 0.01.

In this process, fixed cost per sample equals to 0 and cost per unit sampled is \$4.22, Cost per hour due to nonconformities produced when the process is in-control $C_0 = \$4.2$ and when the process is out-of-control $C_1 = \$420$. Also, cost per false alarm and cost to locate and repair the assignable cause $Y = W = \$977.40$. Other parameters of process are defined as follows:

$E = T_0 = T_1 = 5/60 = 0.083$,$T_2 = 45/60 = 0.75$, $\delta_1 = 1$, $\delta_2 = 0$

Multi-objective economic-statistical design of the CCC control chart in Equation (11) is applied. In this model ARL_L is assumed to be equal to 10000. The upper and lower bounds of decision parameters are defined as:

$2 \leq l \leq 500$ *and* $0 \leq h \leq 2$.

This multi-objective model is solved by the NSGA-II algorithm with the n-pop equals to 50 and optimal values of design parameters are reported in Table 1.

($p_0 = 0.0001$,$p_1 = 0.001$)

The Pareto front for cost and ARL_1 of the multi-objective economic-statistical design of CCC control charts is shown in Figure 3.

h	*l*

Figure 2. Pattern of a chromosome

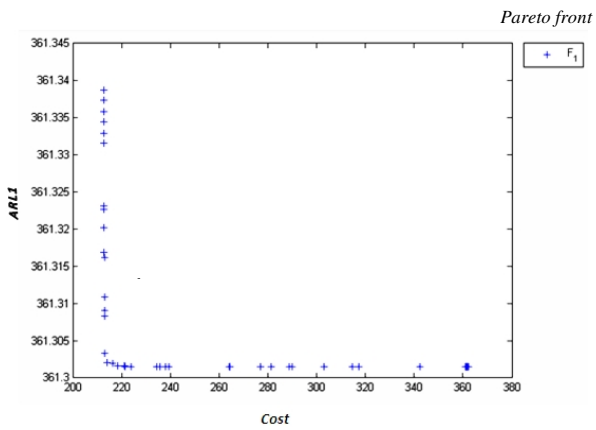

Figure 3. Pareto front for cost and ARL_1

The design parameters of the CCC control chart are optimized by minimizing both the economic cost function and ARL_1. Based on the Table 1 and assuming that the cost function is more important than the other objective function in designing the considered control chart, it can be concluded that the preferred solution from the economic and statistical viewpoint is the solution vector (l, h) = (111.1955, 0.0619). By using $l^* = 111.1955$ and $h^* = 0.0619$, total cost is equal to \$212.7162 and ARL_1 is equal to 361.3387. Also, probability of Type I and II errors obtained equal to 0.01096 and 0.33408, respectively. As discussed in the Introduction section, the economic design of the CCC control chart is investigated by Xie et al. [15]. The result of the preferred solution of our proposed method is compared with the optimal solution of economic design of the CCC control chart in Table 2. As Table 2 shows, the preferred solution of the proposed multi-objective economic-statistical design of CCC control chart is better than the optimal solution of the economic design of CCC control chart reported by Xie et al. [15]. In other words, expected total cost in multi-objective economic-statistical model is less than the expected total cost in the economic design of CCC control chart. In addition, probabilities of Type I and II errors in the proposed model are better than their values in economic model of CCC control chart.

TABLE 1. Pareto optimal solutions of multi-objective economic-statistical model of CCC control charts ($p_0 = 0.0001$,$p_1 = 0.001$)

l	h	C	ARL_1	ARL_0	α	β
111.1955	0.0619	212.7162	361.3387	922552	0.01096	0.334081
111.1918	0.0619	212.7174	361.3358	922583	0.010959	0.334093
111.1786	0.0619	212.7192	361.3345	922691	0.010958	0.334137
111.175	0.0619	212.7201	361.3329	922720	0.010958	0.334149
111.1772	0.0619	212.7218	361.3232	922703	0.010958	0.334142
111.1815	0.062	212.7255	361.3226	922667	0.010958	0.334128
111.1724	0.0621	212.7703	361.3202	922742	0.010957	0.334158
111.1603	0.0623	212.8073	361.3169	922842	0.010956	0.334198
111.1581	0.0626	212.8969	361.3162	922860	0.010956	0.334205
111.1354	0.0626	212.9115	361.3108	923047	0.010954	0.334281
111.1308	0.0628	212.9698	361.3091	923085	0.010953	0.334296
111.1286	0.0632	213.0633	361.3083	923103	0.010953	0.334304
111.1174	0.0635	213.1589	361.3033	923196	0.010952	0.334341
111.1146	0.0661	214.0133	361.302	923219	0.010952	0.33435
111.1146	0.072	216.2709	361.3019	923219	0.010952	0.33435
111.1139	0.077	218.5194	361.3016	923224	0.010952	0.334352
111.1136	0.0825	221.2167	361.3016	923227	0.010952	0.334353
111.1135	0.0827	221.3414	361.3015	923228	0.010951	0.334354
111.1135	0.0874	223.7249	361.3015	923228	0.010951	0.334354
111.1135	0.107	234.2795	361.3015	923228	0.010951	0.334354
111.1135	0.107	234.2874	361.3015	923228	0.010951	0.334354
111.1135	0.1094	235.5637	361.3015	923228	0.010951	0.334354
111.1134	0.1138	237.9599	361.3015	923229	0.010951	0.334354
111.1134	0.1139	238.0006	361.3015	923229	0.010951	0.334354
111.1134	0.1166	239.4538	361.3015	923229	0.010951	0.334354
111.1134	0.1675	264.2473	361.3015	923229	0.010951	0.334354
111.1134	0.1683	264.602	361.3015	923229	0.010951	0.334354
111.1134	0.1988	276.9854	361.3015	923229	0.010951	0.334354
111.1134	0.2106	281.3032	361.3015	923229	0.010951	0.334354
111.1134	0.2328	288.8486	361.3015	923229	0.010951	0.334354
111.1134	0.331	289.8884	361.3015	923229	0.010951	0.334354

TABLE 2. Comparison Multi-objective economic statistical and Economic design of CCC control charts

Multi-objective economic statistical design					Economic design				
212.72	111.19	0.0619	0.0109	0.3341	820.08	962.08	0.0214	0.0916	0.3822

TABLE 3. Sensitivity analysis of multi-objective economic statistical model of CCC control charts under different values of p_0

p_0	p_1	C	ARL_1	l	h	α	β
0.0001	0.001	212.902	383.13	111.04	0.0609	0.011	0.90037
0.0002	0.001	216.192	378.26	105.7009	0.0637	0.02072	0.90054
0.0003	0.001	227.032	362.001	99.36	0.0933	0.02908	0.90628
0.0004	0.001	236.885	363.84	99.002	0.1106	0.03845	0.9066
0.0005	0.001	245.082	360.15	98.65	0.1279	0.04766	0.90692
0.0006	0.001	242.8	358.204	96.64	0.1246	0.05578	0.90875
0.0007	0.001	287.857	357.82	96.48	0.2326	0.06467	0.90888

TABLE 4. Sensitivity analysis of multi-objective economic statistical model of CCC control charts under different values of p_1

p_0	p_1	C	ARL_1	l	h	α	β
0.0001	0.001	418	4105.6	862.08	0.192	0.0825	0.42252
0.0001	0.002	360.9	1789.7	481.75	0.09	0.0464	0.61817
0.0001	0.003	338.3	1193	321.45	0.0873	0.0314	0.7257
0.0001	0.004	305.7	894.6	241.37	0.0755	0.0235	0.78624
0.0001	0.005	275.3	715.6	193.27	0.062	0.0194	0.825
0.0001	0.006	253.9	596.26	161.11	0.0466	0.0158	0.85198

6. SENSITIVITY ANALYSIS

In this section, a sensitivity analysis on the input parameters of the multi-objective model of the CCC control chart is done. There are fifteen input parameters in the Lorenzen and Vance [9]cost function used in the economic-statistical model of CCC control chart. The process parameters consist of p_0 and p_1 which define the in-control and out-of-control states of the process. The effect of these parameters on the best solutions of multi-objective economic-statistical model of the CCC control chart is studied in this section. Moreover, the effect of other input parameters on the best solutions of the proposed model is investigated. Sensitivity analyses on the best solutions of the multi-objective economic-statistical model of the CCC control chart under different values of p_0 and p_1 are done and the results are summarized in Tables 3 and 4, respectively. The other parameters such as the probability of false alarm (α) and the probability of Type II error (β) are also listed in these tables. Table 3 shows that the optimal total cost (C) is an increasing function of p_0. For a given value of p_1, the optimal sampling interval (h) is increasing when the value of p_0 increases. Also, for a given value of p_1, optimal control limit(l) is decreasing when the value of p_0 increases. The results show that the probability of Type I and II errors are increasing when the value of p_0 increases. Also, Table 4 demonstrates that the optimal total cost (C) is an decreasing function of p_1. For a given value of p_0, when the value of $p_1$increases, the optimal sampling interval (h) is increasing and optimal control limit(l) is decreasing. The results show when the value of p_1 increases, the probability of Type I error is decreasing while the probability of Type II error is increasing. Table 5 shows sensitivity analysis of the proposed model under three input parameters. These parameters are including time to sampling and charting a sample (E), cost per false alarm (Y) and λ.

$E\epsilon\ \{0.083, 0.83\}, Y\epsilon\ \{378, 978\}, \lambda\epsilon\ \{0.01, 0.02\}$

Table 5 shows that with increasing value of time to sampling and charting a sample (E), optimal total cost is increasing and ARL_1 is decreasing while optimal sampling interval (h) is increasing and optimal control limit (l) is decreasing. However, changes in the time to sampling and charting a sample (E) has slight effect on the values of probability of Type I and II errors (α and β). In addition, Table 5 shows that changes in the cost per false alarm (Y) has little effect on the optimum values of ARL_1 , probability of Type I and II errors (α and β). When value of the cost per false alarm (Y) is increasing, optimal total cost and control limit (l) is increasing and optimal sampling interval (h) is decreasing. This example shows that the parameters E, Y and λ are the most sensitive input parameters which affect the optimum values of design parameters in the multi-objective economic-statistical model of the CCC control chart.

TABLE 5. Sensitivity analysis of multi-objective economic-statistical design for CCC control charts

E	Y	λ	C	ARL_1	l	h	α	β
0.083	378	0.01	216.19	378.26	105.7	0.102	0.0104	0.90055
		0.02	221.03	362.00	99.36	0.093	0.0098	0.90628
	978	0.01	217.47	374.12	103.97	0.068	0.0102	0.90211
		0.02	224.72	361.33	111.19	0.062	0.0109	0.89562
0.83	378	0.01	218.14	364.13	98.87	0.162	0.0097	0.9067
		0.02	223.09	360.47	97.85	0.095	0.0106	0.89862
	978	0.01	236.88	363.84	99.003	0.11	0.0097	0.90661
		0.02	227.39	360.97	103.22	0.078	0.0101	0.90278

7. CONCLUSION AND FUTURE RESEARCHES

In this paper, a multi-objective economic-statistical model was proposed for designing CCC control chart. The proposed model was solved by a multi-objective algorithm (NSGA-II). A numerical example was used to illustrate effectiveness of the proposed procedure. In addition, the performance of the proposed method is compared with the performance of economic design of the CCC control chart. The results showed that the proposed multi-objective model can address the disadvantages of the economic model. Multi-objective economic-statistical design of the CCC control chart is better than the economical. In the other words, the proposed model leads to better economic and statistical properties. Moreover, a sensitivity analysis is done under different values of input parameters. The results showed that the input parameters including p_0, p_1, E, Y and λ affect the optimum design parameters of multi-objective economic-statistical model of CCC control charts significantly. Multi-objective economic-statistical design of other processes such as multistage processes (Niaki et al. [32], Niaki and Moeinzadeh [33]) or processes with profile quality characteristic (Niaki et al. [34], Keramatpour et al. [35], Abdella et al. [36]) can be considered as future researches.

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Multi-objective Economic-Statistical Design of Cumulative Count of Conforming Control Chart

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چکیده

نمودار های کنترل CCC برای پایش مشخصه های کیفی در فرآیندهای با کیفیت بالا به کار برده می شوند. هزینه عملیاتی نمودارهای کنترل به عنوان محرکی برای محققان در طراحی نمودارها با کمترین هزینه می باشد. در بسیاری از پژوهش ها، تابع هزینه به عنوان یک تابع هدف با در نظر گرفتن محدودیتهای آماری مینیمم می شود که این رویکرد موثری در طراحی اقتصادی-آماری نمودارهای کنترل نمی‌باشد.در این مقاله، یک مدل چند هدفه برای طراحی اقتصادی-آماری نمودار کنترل CCC توسعه داده شده است. سپس الگوریتم تکاملی چندهدفه (NSGA-II) برای بدست آوردن جواب بهینه پارتو مدل پیشنهاد شده است. برای نشان دادن اثر مدل پیشنهادی، از یک مثال عددی استفاده شده است. این مدل اقتصادی - آماری پیشنهادی منجر به هزینه کمتر و احتمالات خطای نوع اول و دوم کوچکتر در مقایسه با مدل اقتصادی می شود. همچنین آنالیز حساسیتی برای بررسی اثر پارامترهای ورودی روی بهترین جواب های مدل پیشنهاد شده انجام شده است.

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