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A note on improved upper bounds on the transversal number of hypergraphs

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A subset T of vertices in a hypergraph H is a transversal if T has a nonempty intersection with every edge of H. The transversal number of H is the minimum size of a transversal in H. A subset S of vertices in a graph G with no isolated vertex, is a total dominating set if every vertex of G is adjacent to a vertex of S. The minimum cardinality of a total dominating set in G is the total domination number of G. In this paper, we obtain a new (improved) probabilistic upper bound for the transversal number of a hypergraph, and a new (improved) probabilistic upper bound for the total domination number of a graph.

Keywords: Hypergraph; transversals; total domination; independence number.

Mathematics Subject Classification 2010: 05C69

1. Introduction

In this paper, we continue the study of transversals in hypergraphs. We obtain a new (improved) probabilistic upper bound for the transversal number of a hypergraph. As a consequence of this result, we obtain a new (improved) probabilistic upper bound for the total domination number of a graph.

A *hypergraph* $H = (V, E)$ is a finite set $V = V(H)$ of elements, called vertices, together with a finite multiset $E = E(H)$ of subsets of $V(H)$, called hyperedges or simply edges. We use the notation $n = n_H = |V|$ and $m = m_H = |E|$. A k-edge in H is an edge of size k. The hypergraph H is said to be *k-uniform* if every edge of H is a k-edge. In the problems studied here, one may assume that $|e| \geq 2$ holds for all edges $e \in E$. The degree of a vertex v in H, denoted by $d_H(v)$, is the number of edges of H which contain v. The minimum and maximum degrees