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Shaped Elevation Pattern Synthesis for Reflector Antenna

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Abstract *In this article, a method based on the invasive weed optimization algorithm is described for synthesizing a point source doubly curved reflector antenna. The proposed method is for designing a reflector with a shaped pattern in an elevation plane and a narrow beam in perpendicular plane. By determining two basic curves, the doubly curved reflector surface can be produced. The central vertical curve approximately produces a shaped elevation pattern. It can be expressed by a well-defined function, and the function parameters will compose a solution space to be explored by invasive weed optimization. The transverse section curve is known to be a part of a parabolic reflector for producing a narrow beam in that plane. The validity of the proposed method is verified by an example of a shaped elevation pattern reflector antenna; the example is a flat-topped elevation pattern. The simulation results based on physical optics further prove the validity and versatility of this technique for solving reflector synthesized problems. Simulation results show good agreement between the defined elevation pattern goal and the achieved elevation pattern. Physical optics simulation during the optimization process makes the synthesis procedure exact and trustable.*

Keywords doubly curved reflector, invasive weed optimization, shaped elevation pattern

1. Introduction

Antennas with shaped patterns in an elevation plane and pencil beams in an azimuth plane are widely used in surveillance–search radar systems. The doubly curved reflector antenna is a classical antenna for this purpose. The synthesis procedure on the basis of geometrical optics (GO) is described in detail by many authors (Silver, 1949; Dunbar, 1948; Milligan, 2005; Brunner, 1971). Different types of configurations (Brunner, 1971; Mizusawa et al., 1978; Karimkashi et al., 2007) and analysis methods for computing its far-field pattern were been presented in Carberry (1969), Winter (1971), Sletten (1981), Pidanic et al. (2008), and Junhao (2006).

The GO method includes GO approximations in the design procedure; thus it is not trustable for many exact applications. Furthermore, the design procedure based on GO is inflexible for achieving extra desired features, such as low sidelobe level or lower ripples at the shaped beam region for antenna performance. Knowing that the doubly curved reflector antenna can be synthesized to produce a shaped beam in one of the principle planes, and after the success of optimization algorithms in antenna problems, led to the

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creation of a new synthesis method for doubly curved reflector antennas. The proposed method could produce any applicable elevation shaped pattern with arbitrary features, and it no longer includes GO approximations.

This study attempts to employ an optimization algorithm to create a synthesis method for doubly curved reflector antennas. The algorithms could find the antenna structure independently, and by defining a proper goal, both synthesizing and optimizing can be done. Describing the central section curvature of a reflector surface using a few parameters and finding the parameters to reach the antenna desired performance are fundamental to the proposed method. It uses physical optics (PO) for analyzing reflector performance during the synthesis process. Therefore, it no longer includes GO approximation and is a more exact design procedure. This method uses invasive weed optimization (IWO) as a tool to perform the idea. IWO is an efficient and robust optimization algorithm method for finding global minima, which was first extracted by Mehrabian and Lucas (2006) and has subsequently been used successfully in antenna design problems (Mehrabian & Lucas, 2006; Mallahzadeh et al., 2008, 2009; Bahreini et al., 2010; Pal et al., 2009).

In this article, the basic concept of GO synthesis is reviewed briefly. The proposed method is then described in detail so that the procedure can be fully understood. In the next step, the example, which is a flat-topped elevation pattern, will show the new method ability in producing any shaped pattern in one plane with different characteristics.

2. Antenna Design Based on GO

For doubly curved reflector antennas, the common way to synthesize a shaped pattern is based on GO (Silver, 1949; Dunbar, 1948; Milligan, 2005). This section provides a review the basic concept of the GO synthesis procedure and extract design formulas.

The doubly curved reflector antenna is made by two main curves. The central vertical curve of the reflector must be designed to provide the desired elevation shaped pattern. The transverse curve must be a part of parabolic reflector; it is a kind of parabola or ellipse for focusing feed rays in the transverse or azimuth plane for producing a narrow beam in that plane (Brunner, 1971).

2.1. *Central Vertical Curve*

In order to shape the reflector antenna's surface to have a prescribed elevation pattern, the proper central vertical section must be found. It means that the central vertical section has the dominant effect on elevation pattern. The central curve is illustrated in Figure 1 (Brunner, 1971). Consider that the phase center of the feed is located at the origin O; φ is the angle of incident ray with respect to z, and θ is the angle of reflected ray. The angle between the incident and reflected rays, σ, can be determined as follows:

$$\sigma = \theta + \varphi, \tag{1}$$

where ρ refers to the distance from the origin to central curve. Clearly, θ and ρ are functions of φ (i.e., $\theta(\varphi)$ and $\rho(\varphi)$). The differential equation of the central curve is (Dunbar, 1948)

$$\frac{d\rho}{\rho d\varphi} = \tan\left(\frac{\sigma}{2}\right), \tag{2}$$

where $\sigma(\varphi)$ and $\rho(\varphi)$ are both unknown. If $\sigma(\varphi)$ is known, $\rho(\varphi)$ can be determined.

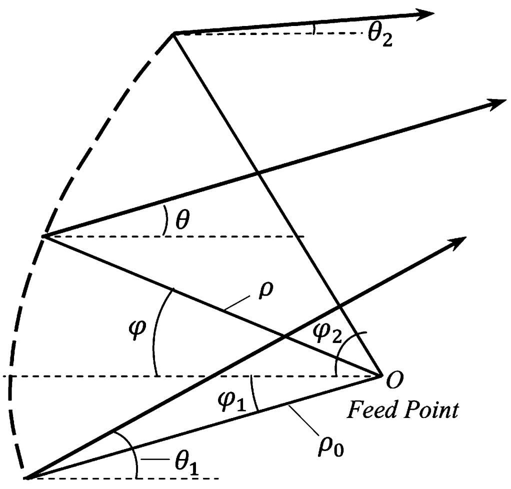

Figure 1. Central vertical curve.

2.2. *Transverse Curve*

The transverse sections of the surface are determined by the requirement that the reflector is to convert a spherical wave into a cylindrical wave. For this purpose, all reflected rays from the reflector in direction θ need to be parallel. Therefore, as illustrated in Figure 2 the following condition must be met (Brunner, 1971):

$$\overline{FP_1} = \overline{FP} + \overline{PA_1}, \tag{3}$$

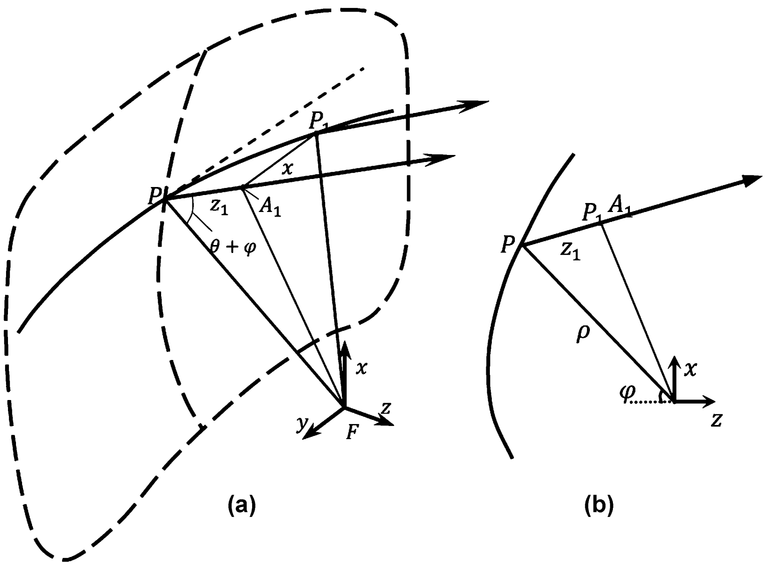

Figure 2. Ray path in transverse section curve: (a) 3D view and (b) side view.

which leads to

$$\left[\rho^2 + z_1^2 - 2\rho z_1 \cos(\varphi + \theta) + y^2\right]^{\frac{1}{2}} = \rho + z_1, \tag{4}$$

The latter equation gives the transverse parabolas of the reflector surface. The desired surface can be achieved by combining both vertical and horizontal curves of the reflector.

To determine the reflector central curve using Eq. (2), it is necessary to derive more useful differential equation based on energy balance principles of GO.

Let $I(\varphi, \psi)$ be the incident energy of feed. $I(\varphi, 0)d\psi d\varphi$ is the power incident on a central element of the reflector. This power is reflected in a wedge with a $\rho d\psi$ width and with a wedge angle $d\theta$. If $P(\theta)$ is the reflected energy of antenna, the following relation can be written based on the GO principle of energy balance (Silver, 1949; Dunbar, 1948; Milligan, 2005; Brunner, 1971):

$$P(\theta)\rho d\psi d\theta = I(\varphi) d\psi d\varphi, \tag{5}$$

Taking the logarithmic derivatives with respect to φ of Eq. (5) and substituting from Eq. (2) gives instead the differential equation

$$\frac{d^2\theta}{d\varphi^2} + \left[\tan\left(\frac{\varphi - \theta}{2}\right) - \frac{I'(\varphi)}{I(\varphi)}\right]\frac{d\theta}{d\varphi} + \frac{P'(\theta)}{P(\theta)}\left(\frac{d\theta}{d\varphi}\right)^2 = 0. \tag{6}$$

This equation cannot be solved analytically and must be solved numerically. Besides, a proper guess for initial value of $d\theta/d\varphi$ is required for the solution process (Silver, 1949). Finding the proper initial value for $d\theta/d\varphi$ needs exhausting trial and error, and different values must be examined to achieve the desired far-field pattern. After finding $\theta(\varphi)$, $\sigma(\varphi)$ and $\rho(\varphi)$ will be clear. By using Eqs. (2) and (4), a whole reflector body is achieved.

For elevation patterns that have an analytical formulation (for example, $\csc^2(\theta)$), $p'(\theta)$ can be derived easily. The solving procedure will face some difficulties when the desired elevation pattern does not have an analytical formulation (for example, a flat-topped pattern).

3. Synthesis Procedure Based on IWO

Recently, employing optimization algorithms, such as the genetic algorithm, particle swarm optimization, ant colony, etc., for solving an antenna optimization problem has been very common. One of the newer algorithms is IWO, which was first proposed by Mehrabian and Lucas (2006), where it was shown to be an effective and robust algorithm to find global minima. According to Mehrabian and Lucas (2006) and Pal et al. (2009), the algorithm process can be summarized as follows.

1. A finite number of seeds are spread out randomly on the search area.
2. They grow to flowering weeds and produce seeds. The number of reproduced seeds of each weed depends on its own fitness, and better fitness permits more seeds to be reproduced. However, the maximum number of seeds is limited.
3. These reproduced seeds disperse over the search area around their parent weeds. The random dispersion has a normal distribution, with mean equal to zero but varying variance (spatial dispersal). The standard deviation (*SD*) decreases in each time step of the algorithm, as in Mehrabian and Lucas (2006), where *iter* is the

number of current time steps, $iter_{\max}$ is the maximum number of iterations, SD_{iter} is the SD at the present time step, $SD_{initial}$ and SD_{final} are prescribed constant values for SD at the first and last iteration, and n is the nonlinear modulation index (usually is set to 3). This decreasing behavior for SD causes aggregation of seeds around better solutions:

$$SD_{iter} = \frac{(iter_{\max} - iter)^n}{(iter_{\max})^n}(SD_{initial} - SD_{final}) + SD_{final}, \tag{7}$$

4. There is a maximum for the number of weeds in each time step, and only plants with better fitness can survive and produce seeds in the next step (competitive exclusion). The process continues until the maximum number of iterations is reached, and finally, the plant with the best fitness is closest to the optimal solution. The flowchart of this algorithm is represented Figure 3 (Mallahzadeh et al., 2008).

3.1. Antenna Synthesis Using IWO

This study employs the ability of IWO for synthesizing a doubly curved reflector antenna. Both antenna synthesis and its optimization are considered here, and IWO will choose the best design depending on its defined goal. The synthesis method has the ability to define the desired pattern and reach toward it, while in synthesis by the GO method, there is not enough flexibility in defining the desired sidelobe level or limited ripple in the shaped region, while PO analysis makes synthesis procedure more exact. The optimization algorithm tries to find the best reflector body in which its characteristics are fitted on the desired characteristics.

It was mentioned in Section 2 that the basic curves of the doubly curved reflector antenna can construct its whole body.

In synthesis procedure based on IWO, the main idea is finding a central section curve that can create a reflector body with the desired shaped elevation pattern. Transverse sections are parabolas and would be determined by knowing the central vertical curve. In order to implement the proposed idea, the following method is recommended.

3.2. Central Curve Synthesis Using IWO

In order to determine the central curve by means of an optimization algorithm, the curve must first be expressed by a finite number of parameters. For example, the curve function can be approximated to an nth order polynomial with $n + 1$ coefficient or parameters. Afterward, these parameters need to be determined properly by means of IWO for creating the central curve. The central curve with the parabolas mentioned in Eq. (4) will create the whole body of reflector. The optimization procedure has a goal that is fitted to the desired shaped elevation pattern.

In this article, the central curve is expressed indirectly. As described in Section 2, if the distribution of $\sigma(\varphi)$ between φ_1 and φ_2 is determined, $\theta(\varphi)$ and $\rho(\varphi)$ will be determined, respectively, according to Eqs. (1) and (2), and then the central vertical curve will be created. For approximating $\sigma(\varphi)$ distribution, various functions were examined to find a function with less parameters and good accuracy. The best choice was a fourth-order polynomial:

$$\sigma(\varphi) = p_1\varphi^4 + p_2\varphi^3 + p_3\varphi^2 + p_4\varphi + p_5, \tag{8}$$

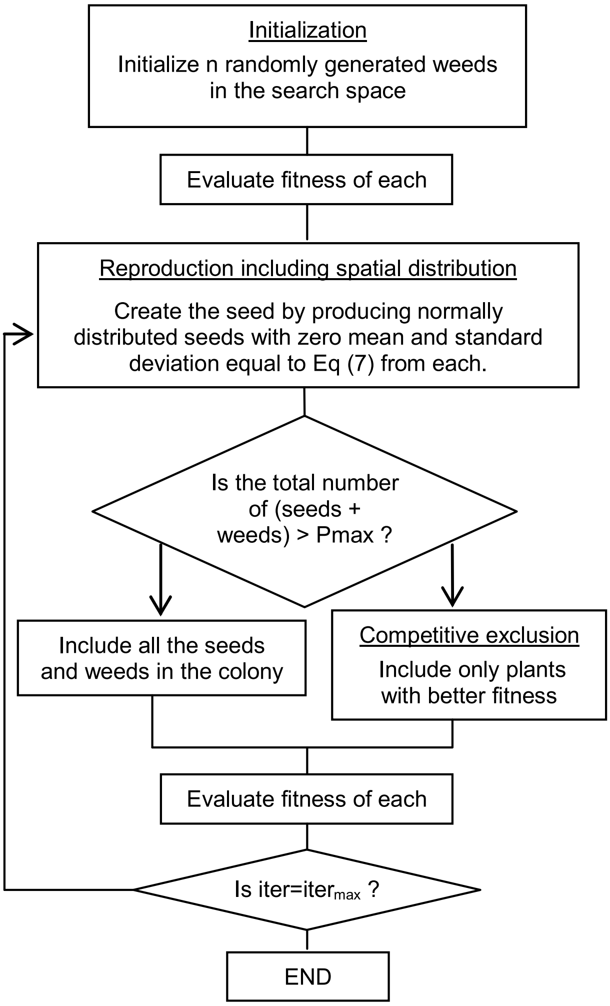

Figure 3. Flowchart representation of IWO algorithm.

where p_i $(i = 1, 2, \ldots, 5)$ denotes the parameters that should be determined. The IWO process starts with initial random coefficients of a chosen function for $\sigma(\varphi)$. Random central curves, and then random surfaces, will be generated according to Eq. (4). So in each time step of the algorithm, there is a number of random reflector surfaces. To obtain the elevation radiation pattern of each reflector, it must be analyzed. Since the radiation pattern of the feed is known, the secondary pattern of reflector can be obtained by means of PO simulations. The obtained elevation pattern is compared with the ideal desired elevation pattern. Consequently, the error (or fitness) value of the weed (produced surface) is the difference between the far-field vertical plane pattern and a desired defined elevation pattern. The optimization process continues until it accomplishes radiation pattern that is closest to the defined elevation pattern.

The important point for any arbitrarily shaped elevation pattern is defining proper goal boundaries and also calculating its error due to these boundaries. The method for calculating the error function depends on each problem's specifications. The example of a shaped elevation pattern is proposed to show the ability of the new method. It should be mentioned that different goal boundaries need to be defined for different examples.

3.3. Flat-Topped Pattern Example

In this section, the desired flat-topped pattern characteristics are described as an example. Two levels of boundaries for the flat-topped pattern are defined (Figure 4). The elevation pattern needs to locate between upper and lower bounds. These limitations force the elevation pattern to become a flat-topped pattern between θ_1 and θ_2 with a sidelobe level as given in Figure 4, $SLL_{desired}$. If the elevation pattern goes beyond these boundaries, the difference between the closer level of boundaries and the achieved pattern is the error at each point of the pattern. Otherwise, the error of the point within the allowable region is zero.

The fitness or error function can be formulized as follows:

$$f = \frac{1}{N}\left(\sum_{-180^\circ}^{180^\circ}\left(\frac{1}{2}(X)^2\right)\right), \tag{9}$$

where

$$X = \begin{cases} 0 & L < G_i(\theta) < U \\ G_i(\theta) - U & G_i(\theta) > U \\ G_i(\theta) - L & G_i(\theta) < L \end{cases} \quad ; \qquad L = \text{lower bound}, \quad U = \text{upper bound}. \tag{9a}$$

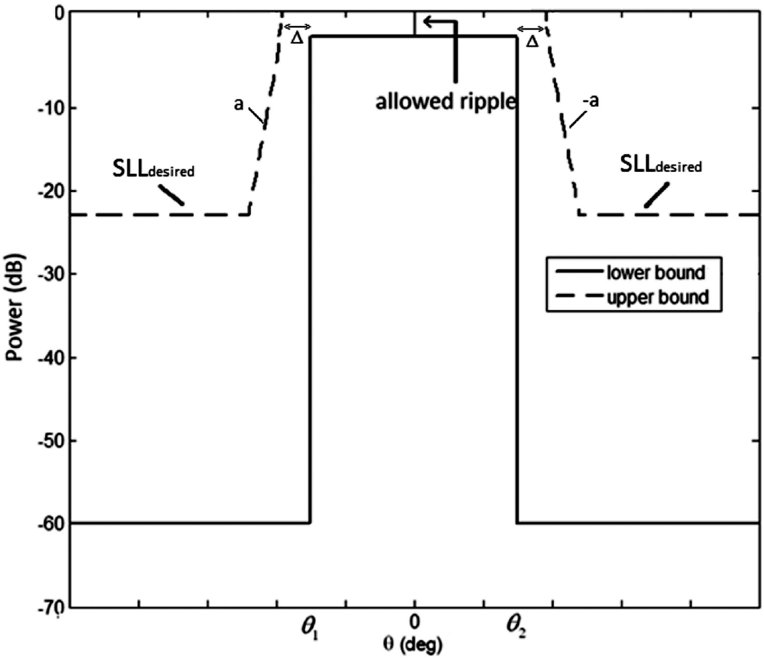

Figure 4. Desired flat-topped elevation pattern boundaries.

Table 1
IWO parameter values

Symbol	Quantity	Value/flat-topped
N	Number of initial population	20
$iter_{\max}$	Maximum number of iterations	25
Dim	Problem dimension	5
$P_{\max}$	Maximum number of plant population	10
$S_{\max}$	Maximum number of seeds	5
$S_{\min}$	Minimum number of seeds	1
N	Non-linear modulation index	3
$SD_{initial}$	Initial value of SD	10
SD_{final}	Final value of SD	0.05
L_{ini}	Initial search area	−2 to +2

4. Simulation Results

In order to show the ability of the IWO algorithm in synthesizing the shaped elevation pattern reflector antenna, detailed simulation results are presented. By choosing an arbitrary dimension for the reflector and designing a suitable feed, the synthesis procedure can be started. For both proposed examples the feed is a typical horn antenna operating at 13 GHz frequency. Since the feed is placed at a 15° offset and has about an 80° 10-dB beam width, φ varies from $-55°$ to $25°$. Therefore, $\sigma(\varphi)$ is specified according to Eq. (8).

By choosing an arbitrary value for ρ_0, the reflector size in the vertical plane would be determined. A 10-dB beam width of feed in the transverse plane and positioning the central section curve in front of the feed specifies the size of the reflector in that plane.

4.1. Simulation Results for Flat-Topped Pattern

IWO parameters for this example are shown in Table 1. The destination elevation pattern needs to have 1.5-dB ripples in the shaped region from −30° to 30° and to have a −23-dB sidelobe level. Therefore the specified parameters shown in Figure 4 are as follows:

$$\theta_2 = -\theta_1 = 30°, \quad \Delta = 8°, \quad \text{and} \quad a = 2.3 \text{ dB/deg} \tag{10}$$

The produced reflectors during the synthesis process and their patterns are shown in Figure 5. It is clear that the final achieved elevation pattern is located between its defined bounds. Variation of coefficients or optimization parameters versus iteration is shown in Figure 6. Final values for the $\sigma(\varphi)$ coefficients that produced the best reflector surface with the desired elevation pattern are shown in Table 2. Figure 7 indicates the convergence of fitness function values. The fitness of produced reflectors are calculated according to Eq. (9).

5. Conclusion

The IWO algorithm, a novel stochastic algorithm, has been successfully employed to create a flexible method to design doubly curved reflector antennas. Total synthesis proce-

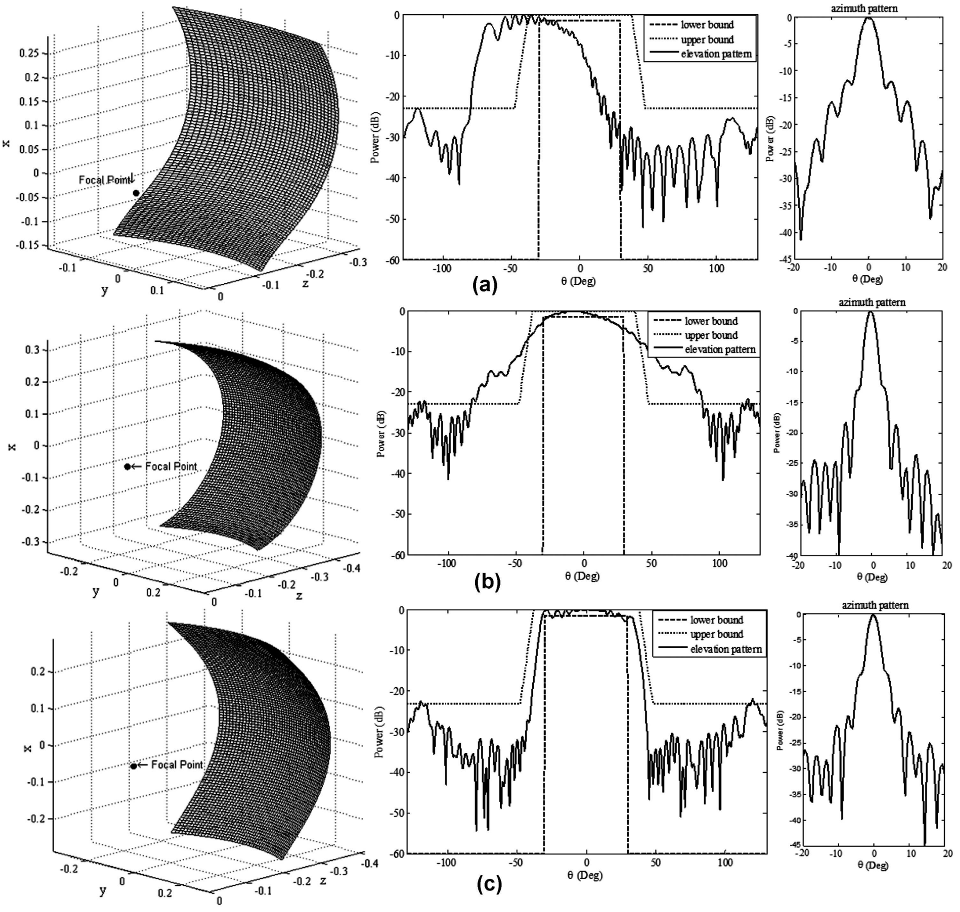

Figure 5. Produced reflector surfaces and their elevation/azimuth pattern for flat-topped pattern example: (a) 1st iteration, (b) 12th iteration, and (c) 25th iteration (final optimum result).

dures based on optimization algorithms can reduce the complexity of the problem and improve the flexibility in design goals and antenna performance. In addition, other antenna parameters, such as positions, orientations, and feed excitation, can be set as other optimization parameters. Furthermore, different types of desired goals can be defined to be achieved. In this article, an example of shaped elevation patterns is synthesized using the proposed method. The results show good agreement between the defined goal and achieved elevation pattern. It shows that the shaped elevation pattern can easily be fitted to its goal. The proposed method is suggested because it gives more flexibility in

Table 2
Optimum values for $\sigma(\varphi)$ coefficient

Value	p_1	p_2	p_3	p_4	p_5
Flat-topped	−0.2676	−1.6715	0.0641	0.6953	−0.0077

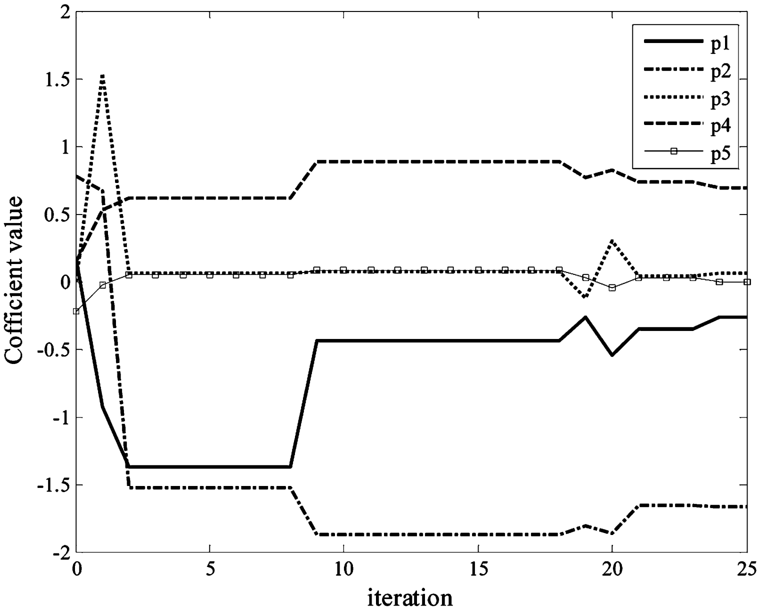

Figure 6. Convergence of fitness value for flat-topped pattern example.

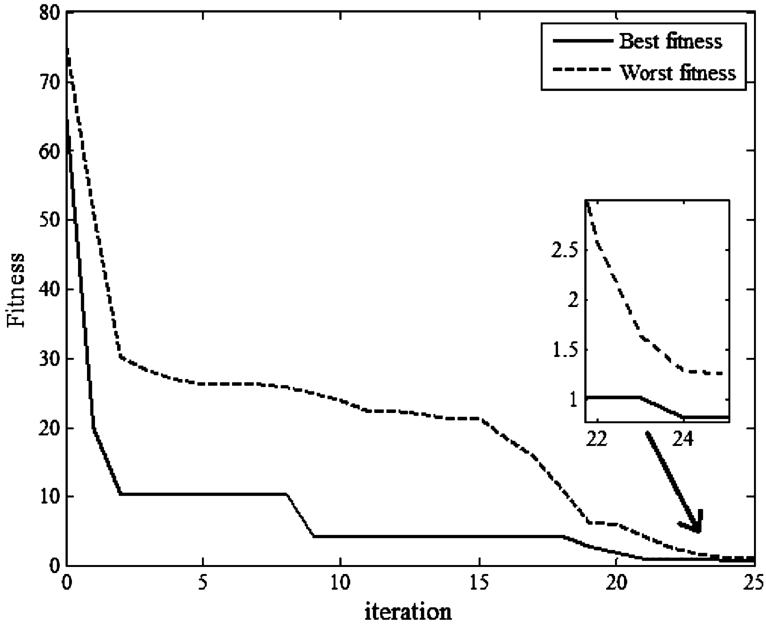

Figure 7. Convergence of coefficient values for flat-topped pattern example.

defining the desirable destination in the pattern. The proposed method can be applied to synthesize any shaped elevation pattern with the desired features, such as width of the shaped region, desired ripple and sidelobe level, and any other definable characteristics in the shape of the elevation pattern.

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