Method for designing low-pass filters with a sharp cut-off

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Abstract: A novel method for designing a compact low-pass filter (LPF) with a sharp cut-off and wide stop-band is presented. In this technique, a simple open-stub microstrip line is printed on top of a substrate and the desired performance is obtained by optimising the shape of the defected ground structure using the genetic algorithm. Details of the design procedure are presented and evaluated through designing two different LPFs. The first LPF is optimised for a 3.5 GHz cut-off frequency and a stop-band up to 10 GHz, whereas the second one is optimised for a 5 GHz cut-off frequency and the stop-band is extended to 16 GHz. Both filters are fabricated and good agreement between simulation and measurement result is obtained. The designed filters have a sharp transition along with a compact size of $25 \times 20 \times 0.787 \text{ mm}^3$.

1 Introduction

Recently, miniature low-pass filters (LPFs) with low insertion loss, sharp transition and wide stop-band have attracted wide interest in wireless communication systems. As is well known, conventional stepped-impedance filters can only provide a gradual cut-off frequency response [1]. A sharp cut-off can be achieved by increasing the filters’ order, although this will intensify the loss in the pass-band and increase the circuit size.

Since the defected ground structures (DGSs) have been introduced [2] and modelled as a lossless parallel LC resonator [3], these structures have been used widely to improve the performance of LPFs by providing a sharp cut-off and an extended stop-band [4–8]. Until now several simple [5, 6] and complex [7, 8] DGSs with different geometrical shapes have been investigated in order to improve the performance of LPFs. In [7], a novel dumbbell-shaped DGS was used for designing an LPF with a sharp cut-off at 4 GHz frequency and wide stop-band from 4.2 to 23 GHz. However, the dimensions of the final structure are about $11\lambda_g \times 4\lambda_g$ (where $\lambda_g$ is the guide wavelength at cut-off frequency) and are not that compact. In [8], a $5.49\lambda_g \times 3.66\lambda_g$ LPF with 3 GHz cut-off frequency and a wide stop-band from 3.2 to 18 GHz was reported. It should be noted that in all the above studies, the authors have not mentioned any design procedures for the proposed DGSs.

Recently, genetic algorithms (GAs) have been used widely for optimising the shape of passive high-frequency components to meet desired frequency responses [9–13]. Thus far GAs have been used to generate non-conventional wire-based [9], broadband patch-based [10] and monopole-based [11, 12] antennas. However, only a few attempts have been made at developing a GA capable of shape optimisation in filter designs. In [13], to obtain a band-pass filter, arbitrarily shaped conductor patches and slots are designed and optimised through a GA. However, the optimisation is done in a very limited frequency range and the final structure has neither a wide stop-band nor a sharp cut-off in the frequency response. In addition, because of optimisation in both sides of the substrate, the fabricated filter has a high radiation loss and cannot be used without a metallic box.

Here, a novel method for designing a compact LPF with a sharp cut-off and wide stop-band is presented. The technique is based on optimising the shape of the DGS at the bottom layer of a predefined structure. The DGS area in the ground plane is divided into many overlapping sub-patches. The presence or non-presence of each sub-patch is determined by a conventional GA and sharp cut-off LPFs with wide stop-band are obtained from the GA. The designed filters can be easily integrated with different types of microwave circuits.

Details of the design procedure with a description of the GA parameters settings and fitness functions are presented in Section 2. Two different LPFs are designed and fabricated in Section 3 in order to validate the proposed algorithm. Measurement results show that the presented design method improves different aspects of the electrical performances of LPFs.

2 Design and optimisation procedures

The proposed technique of designing compact LPFs with a sharp cut-off is based on using a conventional GA in order to create a non-intuitive shape of the DGS at the bottom layer of a predefined structure.

A perspective view of this predefined structure is shown in Fig. 1. As shown, this structure consists of an open-stub
microstrip line on top of the substrate and a rectangular DGS area at the other side. The dimensions of the open-stub microstrip line are \( W_p = 4.25 \text{ mm} \) and \( L_p = 8.25 \text{ mm} \), which is connected to a 50 \( \Omega \) line. The rectangular DGS area on the ground plane with \( L_{\text{DGS}} = 15 \text{ mm} \) and \( W_{\text{DGS}} = 13 \text{ mm} \) can be filled by many square sub-patches of 1 mm \( \times \) 1 mm. Each of these sub-patches at the ground area can be switched ‘on’ or ‘off’ by the GA. To ensure electrical contact in this configuration, sub-patches overlap each other by \( L_0 = 0.05 \text{ mm} \), as shown in Fig. 2.

The predefined structure is printed on a Rogers 4003 substrate with a thickness of 0.787 mm, relative constant of 3.55 and dielectric loss tangent of 0.0027. The studied filter has compact dimensions of 20 \( \times \) 25 \( \times \) 0.787 mm\(^3\).

As is well known that a GA optimises a string of bits known as a chromosome. A chromosome is composed of individual binary coded genes that represent the structure parameters. Here, gene at the chromosome is encoded into the presence or absence of sub-patches in the DGS area. In each bit of the gene, a ‘1’ represents a cell with metal and a ‘0’ represents a cell with no metal.

Unlike previous studies, in which the presence or absence of all the sub-patches are represented by only one gene value [8–11], in this study each column of the DGS area is represented by a specified gene. Thus the studied chromosome consists of 13 binary coded genes each having a search space of 0–2\(^{13}\).

The GA is initialised with a group of randomly generated chromosomes. Each of the generated chromosomes consists of 13 different genes. These genes are then encoded to the presence or absence of sub-patches at the related column of the DGS area, as shown in Fig. 2, and create a non-intuitive shape of the DGS on the bottom layer of the predefined structure. The fitness value of each arbitrarily created DGS shape is then evaluated by a cost function.

Each new generation is obtained from the previous one. The fittest individuals may be passed unchanged to the next generation. Cross-over and mutation are also executed on the fitter individuals in order to replenish the population to its original size. In cross-over, a new chromosome is obtained by combining a pair of tournament winners, while in mutation random bits of chromosomes are changed. This iterative process goes on until a predefined stop criterion is reached. Owing to its robust stochastic search method, it is expected that this automotive iterative process converges to the global maximum. A flowchart of the design and optimisation procedure is shown in Fig. 3. The GA is implemented in MATLAB and the generated DGS shape is sent to a commercial finite element method (FEM) solver, HFSS, through visual basic scripts.

In Fig. 4, a schematic model for filters that can be obtained from the DGS optimised by GA is shown. The open-stub microstrip line on top of the substrate acts as a first-order stepped-impedance filter with a very gradual transition [1]. Furthermore, because of the band-stop characteristics of the DGSs, they are usually modelled by a parallel LC resonator [3]. Therefore we can model the DGS of the proposed structures by many series LC resonators, representing the presence of sub-patches at the ground plane, while coupling between the open-stub microstrip line and the sub-patches is modelled by parallel capacitors \((C_k)\) [3, 4].

It should be noted that because of low-level coupling between the DGS resonators at the ground plane, the GA could not converge very well to an LPF without using an open-stub microstrip line on top of the substrate as a compensator.

### 2.1 Definition of the cost function

As it is described in the previous section, in order to evaluate the fitness of each generated chromosome a cost function should be defined. The cost function is defined by

\[
F = \sum_{i=1}^{N} \left[ w_{f_1}(E_{1(k)})^2 + w_{f_2}(E_{2(k)})^2 \right]
\]

in which

\[
E_k(f_i) = |S_1 k(f_i)|_{\text{results}} - |S_1 k(f_i)|_{\text{desired}}
\]

\(N\) is the number of sampling frequencies, \( f_i \) is the \( i \)th sampling frequency, \( w_{K_i} \) represents the weighting value at the \( i \)th sampling frequency and \( E_k \) is the difference between the magnitude of the calculated scattering parameter and the desired scattering parameter, as shown in (2). It should be mentioned that because of the radiation loss caused by the DGS at the ground plane, the error of \( S_{11} \) and \( S_{12} \) from the desired values should be calculated in the cost function simultaneously. Furthermore, in order to have a better
convergence in the GA, we should neglect small errors in the sampling frequencies. This is done by defining some criteria for the weighting values as follows

for the pass-band samples:
\[
\begin{align*}
    w_{1i} &= 0 & \text{if } |S_{11}| &< 0.95 \\
    w_{2i} &= 0 & \text{if } |S_{21}| &< 0.05
\end{align*}
\]  

(3)

for the stop-band samples:
\[
\begin{align*}
    w_{1i} &= 0 & \text{if } |S_{11}| &\geq 0.95 \\
    w_{2i} &= 0 & \text{if } |S_{21}| &\geq 0.05
\end{align*}
\]  

(4)

For having a sharp cut-off, these criteria are ignored at the sampling frequencies around the cut-off frequency. The \( F \) value evaluated by (1) is allocated to each generated chromosome as a fitness value.

3 Design examples

Here, two compact LPFs with wide stop-bands and sharp cut-offs are designed and implemented to evaluate the proposed algorithm. The first filter is designed and optimised for a 3.5 GHz cut-off frequency and a stop-band up to 10 GHz, and the second one is designed and optimised for a 5 GHz cut-off frequency and the stop-band is extended to 16 GHz.

Both of the proposed filters have the same dimensions and structures as mentioned in the previous sections and only the DGS schemes, obtained by GA optimisation, differ from one another. Thus, this technique can be used to design any LPFs with a sharp transition and desired cut-off frequency. Details of the design procedure of each filter are explained in the following sections.
3.1 Design procedure and measurement results of the LPF with a 3.5 GHz cut-off frequency

The desired scattering parameters of the first LPF designed by GA, with the weighting values at each frequency region are shown in Fig. 5. The DGS scheme of the first filter is designed and optimised to have a 3.5 GHz cut-off frequency and a stop-band up to 10 GHz. The weighting values are carefully changed in the pass-band, the transition region and the stop-band to obtain a better frequency response. The weighting values for the pass-band (0.5–3.4 GHz), the stop-band (4.2–10 GHz), the transition region (3.4–3.6 GHz) and right after the transition region (3.6–4.2 GHz) are set to 1, 0.5, 0 and 3, respectively.

These weighting values would help the algorithm to converge into a sharp cut-off rather than smooth ones. Also small errors in frequency samples of the pass-band and stop-band are ignored by zeroing the related weighting values as described in Section 2.2.

The population size of the GA is set to 40 individual chromosomes. About 65% of the chromosomes are obtained from the cross-over function, 15% of them are obtained from the mutation function and the rest of them are the ones that migrated from the previous generation to the next one. The cost function is evaluated in 96 frequency samples for each individual chromosome.

The scattering parameter of the designed filter at different stages is shown in Fig. 6. Referring to that figure, because of the use of open-stub microstrip line on top of the substrate, the initial randomly generated DGS have a very gradual cut-off response and very limited stop-band.

However, at the final stage, DGS sub-patches are changed and an LPF with sharp cut-off and wide stop-band is obtained.

The convergence plot of the best fitness value is shown in Fig. 7. As shown, after passing 50 generations the algorithm terminates. A single run of HFSS on one trial solution filter takes approximately 1 min using a computer with a Core i7–2.5 GHz CPU. Thus for the given population size and generations, the simulations in HFSS take about 33 h. Finally, the best chromosome has a fitness value of 3.4 evaluated by (1). The created DGS shape of the best chromosome was previously shown in Fig. 2. The fabricated prototype of the optimised LPF with a 3.5 GHz cut-off frequency is shown in Fig. 8.

The simulation and measurement results of the proposed LPF are shown in Fig. 8. Referring to that figure, the return losses are larger than 12 dB and the insertion losses are...
smaller than 1.8 dB from DC to 3.4 GHz. The designed low-pass filter also exhibits a wide stop-band with a rejection better than 15 dB from 4.2 to 13 GHz. It can also be seen that the non-initiative DGS shape provides a transmission zero at 4 GHz with almost 40 dB of rejection. The proposed filter has a good skirt response with a selectivity of 43 dB/GHz.

From the frequency response of the designed filter, some notches are seen in both simulated and measured $|S_{11}|$ around 11 GHz, without significant changes in $|S_{12}|$. These notches are because of radiation loss from the designed filter at higher frequencies. Although these losses do not affect the performance of the designed filter, they may cause interference with other systems. Therefore in some applications it is needed to package the fabricated filter into a metallic box. The effect of the metallic box on the performance of the designed filter will be discussed in Section 4.

3.2 Design procedure and measurement results of the LPF with a 5 GHz cut-off frequency

For the second example, we try to design and optimise the DGS shape of the filter in order to have an LPF with a wider pass-band and stop-band. As shown in Fig. 9, we have chosen 5 GHz as the cut-off frequency and a stop-band up to 16 GHz. Similar to the previously designed filter, the weighting values for the second LPF are changed in different frequency regions and are chosen as 1, 0, 3 and 0.5, for the pass-band (1–4.9 GHz), the transition region (4.9–5.1 GHz), right after the transition region (5.1–6 GHz) and the stop-band (6–16 GHz), respectively.

The population size of the created generations is set to 40 and the cross-over and mutation rates are set to 0.65 and 0.15, respectively. For each simulation run, the fitness value is evaluated in 156 frequency samples from 0.5 to 16 GHz (every 100 MHz).

The scattering parameter of the designed filter at different iterations is shown in Fig. 10 and the convergence plot of the best chromosomes in each generation is shown in Fig. 11. In comparison with the first LPF, more frequency samples are needed for fitness evaluation. Thus, the stop criterion for the GA run is extended to the 90th generation in order to have an acceptable fitness in the end.

Each single run of the proposed filter takes approximately 1.5 min using a computer with a Core i7-2.5 GHz CPU, thus the algorithm stops after 90 h run on a single machine. The best chromosome in the 90th generation has a fitness value of 3.3 evaluated by (1). The created DGS shape of the best chromosome is shown in Fig. 12a. The top view of the fabricated prototype of the 5 GHz LPF is similar to 3.5 GHz LPF and is shown in Fig. 8a. The bottom view of the fabricated prototype is shown in Fig. 12b. The measurement and simulation results of the best chromosome are compared in Fig. 12c. The proposed filter has a 3 dB cut-off frequency at exactly 5 GHz. The return losses are larger than 15 dB and the insertion losses are smaller than 0.8 dB in the pass-band frequencies. The filter also has an ultra-wide stop-band with a rejection almost better than 20 dB from 5.2 to 20 GHz. A transmission zero with a rejection of 40 dB can be realised about 5.8 GHz. The selectivity of the proposed DGS filter is about 44 dB/GHz. Thus, the proposed filter has an ultra-wide stop-band and a sharp transition response simultaneously.

Similar to the previous LPF, some notches are observed in the simulated and measured results of $|S_{11}|$ about 18 GHz. This is again because of radiation loss from the designed filter and can be eliminated by a metallic box as described in the following section.

It should be mentioned that the second LPF obtained from the GA optimisation has more compact dimensions and wider 3-dB fractional bandwidth compared with the previously reported DGS LPFs [7, 8].
Fig. 13 shows the effect of metallic enclosure height on the frequency response of the 5 GHz LPF. (The same effect can be observed for the 3.5 GHz cut-off LPF.) It can be seen that as the distance $d$ between the metallic box and the filter ground decreases, the insertion loss in the pass-band increases. Referring to Fig. 13, a metallic box with almost 5 mm height from the ground of the filter may result in an appropriate frequency response in both pass-band and stop-band frequencies.

5 Conclusions

A novel method for designing compact LPFs with a sharp cut-off and wide stop-band has been presented. It was shown that an LPF with a desired cut-off frequency and sharp transition can be realised by printing a microstrip open-stub on top of a substrate and optimising the DGS shape on the bottom layer by a GA. To evaluate the proposed algorithm, two different LPFs with 3.5 and 5 GHz cut-off frequencies have been designed and implemented. The measurement results of both compact filters have shown sharp cut-offs and wide stop-bands in the filters’ frequency responses. Furthermore, the second filter has a wider 3-dB fractional bandwidth compared with that of recently developed LPFs based on DGS. Therefore this technique can be easily implemented to design different types of LPFs.

6 References