

Comparison of Two Goal Programming Models for Budget Allocation Problem - A Case Study

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Abstract

Each organization has an array of major goals. At a superficial look, they seem coordinated and congruent but when managers break them down into smaller goals and try to allocate the budget of each of them, multiple conflicting goals pop up and make budget planning seriously challenging. Conflict among organizational goals is a common feature of multi-objective entities and causes an imbalance in achievement of the set of goals. Namely, one goal is achieved at the expense of deviation or less achievement of another goal. Because organizations such as universities, municipalities, financial and credit corporations and industrial firms are multi-objective systems, they are not an exception to imbalanced achievement of goals.

The imbalanced achievement of goals has a very dysfunctional effect on organization's all-out performance. To avoid this, all budgeters and resource allocators should be aware of efficient techniques for budget planning in multi-objective systems. Since multi-objective nature of organizational systems endogenously brings about an amount of imbalance over achieving the goals, reducing this imbalance is always regarded as a very important problem for OR experts so that many efforts have been made to solve it up to now. Generally, many mathematical models have been formulated for this purpose from which the Goal programming (GP) seems to be the most promising one.

Unlike Linear Programming (LP), GP is multi-objective and different variants of its have been developed for multi-objective problems, among which Lexicographic Goal Programming (LGP) Weighted Goal Programming (IWGP) have been frequently used for budget planning. But, because LGP and WGP fail in achieving a good balance between the achievements of the set of goals, especially when decision makers like their goals to be achieved by a good balance, a better alternative needs to be used. The Chebyshev goal Programming (CGP) is what this paper proposes as the most appropriate method when the decision makers like to observe a good balance among their achieved goals. Thus, supported by a real world example, this paper compares GP models in terms of balancing principle and shows their potentials to achieve a good balance.

1. Introduction

In the era of complexity, organizations have to carry out a range of activities in order to attain their major goals. These activities can in turn be defined as sub-goals which have to be constantly pursued if organizations want to attain their major goals. But, pursuing sub-goals can be often more challenging than expected. To realize it better, suppose a toy manufacturing company as an example when its CEO is decided to achieve the 20 percent of market share by next five years. To do so, a number of sub-goals have been set entailing (I) selection of capable human resources, (II) Human resource training and development, (III) Organizational structure re-engineering, (IV) Cost reduction, (V) Quality enhancement and (VI) Customer relationship Management. Up to this point, everything seems right but as soon as managers of organizational divisions start pursuing these sub-goals, a problem comes into view. In simpler terms, this problem comes directly from the fact that such sub-goals are unaligned and can't be attained without considering their trade-offs. To make it clearer, suppose a university as another example where, (I) knowledge dissemination, (II) human resources Training for the preservation and development of the nation and society, (III) researchers Training and preparation for preserving and expanding the human knowledge, and (IV) the cultivation and enhancement of the individuals' inner potentialities are believed to be the four main goals that at least all universities pursue to gain [13]. By a glimpse, these major goals seem coordinated and congruent but while being divided into sub-goals and considered for budget planning, several contradictory goals rise, for example, the goal of offering a high educational service as a sub-goal of goal I and the goal of building sophisticated laboratories as a sub-goal of goal II seem somewhat contradictory when managers plan to divide the budget between them.

Nevertheless, multi-objective nature of socio-economic systems has made budget planners faced with some budget allocation challenges among which the unbalanced allocation of budget among organization's multiple goals is believed to be the most important one. To surmount this challenge, budget planners need to be informed of powerful budget allocation methods. To do so, different budget allocation methods have been developed among which the goal programming (GP) methods seem to be widely applied. But, GP methods in turn have different performances in terms of balanced allocation of Budget among goals. So, by comparing two widely applied GP models in real case study, this paper is aimed at selecting the most appropriate GP model for balanced allocation of budget among organizational Goals.

2. Application of GP in budget planning

In 1987, using a survey of 146 articles, White [5] showed that the available models can be implemented in higher education administration. In a research conducted in 2001, Romero and caballero emphasized the application of quantitative models for solving resource allocation-related problems of the universities. These two researchers in a study that conducted in 2006 could design an interactive goal programming model. By

this model, managers could overcome so many of the resource allocation challenges. One of the outstanding features of this model is that it enabled managers to cope with the unexpected and uncertain environmental phenomena smartly [6]. Hopkins developed a cost simulation model in which the budget was considered as an output of the model rather than input [2]. On the other hand, Schroeder designed a model in which the budget for future planning years was taken into account as an input [4]. Basu and Pal used a goal programming model for allocating the budget within the existing academic units in a university in future planning period, their model was able to allocated the budget for attainment of the desired level of teaching staff, non-teaching staff and research fellows [1]. Nopiah and associates developed a comprehensive model for university budget planning. The comprehensiveness of this model empowered planners to cover different parts of an educational system and track the resource allocation flow more precisely [7]. Inspired by the work of Nopiah and associates, Safari and associates developed an Integer Lexicographic Goal Program. This model possessed 36 variables, 49 goals, 7 systematic constraints and 53 technical coefficients. After it had been solved, the goals were achieved with a total deviation (I. e. Objective Function Value) equal to 250.6875 units. [10]. Zamfirescu and Bela Zamfirescu used a goal programming model for performance-based budgeting. In this research, they developed a spreadsheet solution in order to partially implement the principles of performance-based budgeting in a public body [11]. Using goal programming and analytic hierarchy process, Tang and Chang developed a model for Capital budgeting investment in a small car rental company. Clarifying the effects of multiple criteria decision making on a capital budgeting investment was their purpose in conducting this research [12].

In a long-range research conducted about university resource allocation systems, pal and Sen could develop an efficient goal programming model for resource right allocation. This model has considered the resources trade-off in the educational systems so well [8]. Dylan Jones also developed a new pattern for sensitivity analysis of resource allocation goal programming models in his studies [9]. Jones and Tamiz have represented the goal programming growing trend up to the late 2000's as the figure 1.

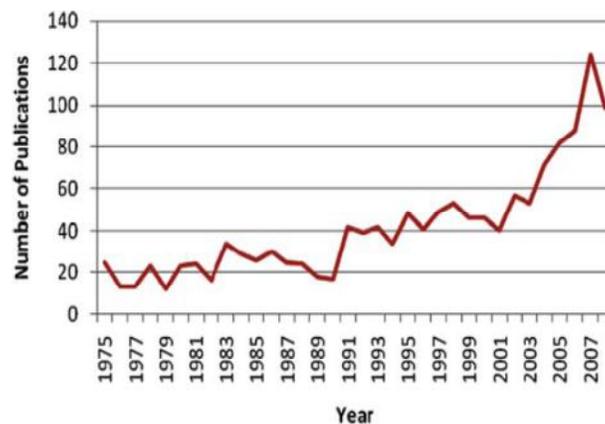


Figure 1. Goal programming publications in period 1975-2008, [3].

However, the problem of unbalanced allocation of budget has still been a basic problem for which this paper is going to present a solution.

3. Different models of GP

GP models can generally be defined in terms of underlying distance metric and mathematical nature of the decision variables and/or goals used in the program. Lexicographic, Weighted, and Chebyshev models are three major models which are defined in terms of underlying distance metric while fuzzy, integer, binary, and fractional goal programming models are defined in terms of mathematical nature of the goals and/or decision variables [3]. Although these two model classes are fundamentally different, it is possible to formulate a goal programming that has a model from each class. Depending on the structure of problem, each of these models may be used. In terms of distance metric, Weighted and Chebyshev models are more applied than Lexicographic one and science variables of problem under study are integer, Integer Weighted Goal Programming (IWGP) and Integer Chebyshev Goal Programming (ICGP) are formulated in this paper.

According to Jones and Tamiz [3], All GP models have four underlying Philosophies. Hence, if organization planners are going to utilize the GP models properly, they have to understand these philosophies thoroughly. These philosophies are the foundations of GP models which include (I) Satisficing, (II) Optimizing, (III) Ordering or ranking and (IV) Balancing. Among these major philosophies, the fourth one is completely related to the problem of unbalanced allocation. Therefore, it has been used as an indicator for comparing ICGP and IWGP in terms of a balanced allocation of budget among organizational multiple goals.

4. Comparison of ICGP and IWGP based on balancing indicator

As Tamiz and Jones say (2010), a common feature in all of the solutions found by both the weighted and lexicographic variants can be found. That is, they are all to be found at extreme points. This leads to a certain imbalance, with some goals doing very well but other goals being a long way from being achieved. This is due to the underlying Manhattan metric that has a 'ruthless optimization' property. If a balance between the objectives is the dominant need, then the Chebyshev Goal Programming (CGP) should be applied. This variant of Goal Programming was first introduced by Flavell. It is known as Chebyshev because it uses the underlying Chebyshev (L_∞) means of measuring distance. That is, the maximal deviation from any goal, as opposed to the sum of all deviations, is minimized. For this reason CGP is sometimes termed Minmax goal programming.

With regard to the problem of unbalanced allocation of budget among organization's multiple goals, the underlying philosophy, when using the L_∞ distance metric, is that of balance. That is, the decision maker is trying to achieve a good balance between the achievement of the set of goals as opposed to the lexicographic approach which deliberately prioritizes some goals over others or the weighted approach which chooses the set of decision variable values which together make the achievement function lowest, sometimes at the expense of a very poor value in one or two

of the goals. CGP is also the only major variant that can find optimal solutions for linear models that are not located at extreme points in decision space.

Based on aforementioned information, it can be concluded that CGP has the potential to give the most appropriate solution where a balance among the levels of satisfaction of the goals is needed. This model can apply to a large number of areas, especially those with multiple decision makers each of whom has a preference to their own sub-goals that they regard as most important. Budget allocation problem as a problem which needs to cover multiple goals is really needful of being formulated by this method. However, this paper is aimed at selecting the most appropriate GP model for balanced allocation of budget among organizational goals. To do so, CGP and WGP as two of the most widely used GP models are compared.

5. Mathematical structure of WGP and CGP

The algebraic structure of WGP and CGP are represented in Figure 2 and 3 respectively.

$$\begin{aligned} \text{Min } a &= \sum_{q=1}^Q \left(\frac{u_q p_q}{k_q} + \frac{v_q n_q}{k_q} \right) \\ f_q(\underline{x}) + n_q - p_q &= b_q \quad q=1, \dots, Q \\ \underline{x} &\in F \\ n_q, p_q &\geq 0 \quad q=1, \dots, Q \end{aligned}$$

Figure 2. Algebraic structure of WGP [3]

$$\begin{aligned} \text{Min } a &= \lambda \\ f_q(\underline{x}) + n_q - p_q &= b_q \quad q=1, \dots, Q \\ \frac{u_q n_q}{k_q} + \frac{v_q p_q}{k_q} &\leq \lambda \quad q = 1, \dots, Q \\ \underline{x} &\in F \\ n_q, p_q &\geq 0 \quad q=1, \dots, Q \end{aligned}$$

Figure 3. Algebraic structure of CGP [3]

Where

λ is the maximal deviation from amongst the set of goals

\underline{x} is the vector of decision variables, $\underline{x} = x_1, \dots, x_n$

$f_q(\underline{x})$ is the achieved value of qth goal

n_q is the negative deviational variable of qth goal

p_q is the positive deviational variable of qth goal

b_q is the aspiration level of qth goal

u_q is preferential weight associated with the penalization of n_q

v_q is preferential weight associated with the penalization of p_q

k_q is normalization constant

F is the feasible set

Furthermore, since the nature of problem under study requires decision variables to take only a discrete number of values within their defined ranges, the model decision variables are written in the integer form..

6. Problem description

An Appropriated Budget of a municipality is 3000 Million Tomans. The organization wants to allocate this budget to expansion of urban Sport facilities. Citizens have called for 5 Sport facilities including Basketball field, Football field, Tennis ball field, Swimming pool and Wrestling saloon. The demand of citizens for each of these facilities is 6, 4, 10, 12 and 7 respectively. So, the decision variables of this problem can be represented in Table 1.

Table 1. Decision variables of problem

Decision variables	
Symbol	Description
x1	The number of basketball fields
x2	The number of football fields
x3	The number of tennis ball fields
x4	The number of swimming pools
x5	The number of wrestling saloons

Based on basic information of sport facilities which is represented in Table 2, the organization's planners have set 9 organizational goals which are shown in Table 3 with their corresponding mathematical formulation.

Table 2. Basic information of sport facilities

Facility	Cost (Million Tomans)	Area (Hectare)	Average of users (person per week)
Basketball Field	250	3	600
Football Field	100	10	1200
Tennis ball Field	50	2	500
Swimming Pool	200	2	1000
Wrestling Saloon	500	3	800

Table 3. Organizational goals and their corresponding mathematical formulation

Number of goal	Description of goal	Mathematical formulation of goal
1	The Organization wants the allocated budget to Sport facilities to be at least 3000 Million Tomans	$250*x1+100*x2+50*x3+200*x4+ 500*x5+n1-p1= 3000;$
2	The organization wants the number of users to be at least 10000 persons per week	$600*x1+1200*x2+500*x3+1000*x4+800*x5+n2-p2=10000;$
3	The Organization wants the total area of all fields not to exceed 45 hectares	$3*x1+10*x2+2*x3+2*x4+3*x5+n3-p3=45;$
4	The organization wants the additional land for expanding the total area not to exceed 10 hectares	$3*x1+10*x2+2*x3+2*x4+3*x5-45+n4-p4=10;$
5	The Organization wants the number of Basketball fields to be at least 6	$x1+n5-p5=6;$
8	The Organization wants the number of Football fields to be at least 4	$x2+n6-p6=4;$
7	The Organization wants the number of Tennis fields to be at least 10	$x3+n7-p7=10;$
8	The Organization wants the number of Swimming pools to be at least 12	$x4+n8-p8=12;$
9	The Organization wants the number of Wrestling Saloons to be at least 7	$x5+n9-p9=7;$

The organization has also two systematic constraints that must be satisfied through the allocation process. These constraints are represented as Table 4

Table 4. Problem's constraints

Number of goal	Description of goal	Mathematical formulation of goal
1	The number of Sport facilities should be at least 14	$x1+x2+x3+x4+x5 \geq 14;$
2	The cost of facilities expansion should not exceed 3000 Million Tomans	$250*x1+100*x2+50*x3+200*x4+500*x5 \leq 3000;$

7. Model designing

After decision variables and hard constraints are identified and goals and their target levels are determined, ICGP and IWGP are separately designed and formulated in LINGO 11.0 software. It is important to note that all preferential weights are set as 1 and normalization constant is computed by Euclidean Normalization Method. However, since every model possesses specific statistical features. The features of ICGP and IWGP are represented in Table 5 and Table 6 respectively.

Table 5. Statistical features of ICGP

Integer variables number	Rows number	9	Goal constraints	5	Decision variables	Integer Chebyshev Goal Programming	Type
5	21	2	Hard constrains	25	Total variables	MINMAX	Direction

Table 6. Statistical features of IWGP

Integer variables number	Rows number	9	Goal constraints	5	Decision variables	Integer Weighted Goal Programming	Type
5	12						
		2	Hard constrains	23	Total variables	MIN	Direction

8. Model's results

After solving the models, the result of each model is represented in Table 7 and Table 8 respectively.

Table 7. Results of IWGP model

Goal number	Target level	Satisfaction	Achieved level	Percentage of deviation
1	3000	YES	3000	0%
2	10000	NO	9000	0.10% under achieved
3	45	YES	45	0%
7	10	YES	10	0%
5	6	NO	0	100% under achieved
6	4	NO	0	100% under achieved
7	10	YES	10	%0
8	12	NO	0	100% under achieved
9	7	NO	5	28% under achieved

Table 8. Results of ICGP model

Goal number	Target level	Satisfaction	Achieved level	Percentage of deviation
1	3000	YES	3000	0%
2	10000	YES	10000	0%
3	45	NO	51	13.3% overachieved
7	10	YES	10	0%
5	6	NO	3	50% under achieved
6	4	NO	1	75% under achieved
7	10	NO	7	30% under achieved
8	12	NO	9	25% under achieved
9	7	NO	0	100 % under achieved

In the above tables, when a target level is completely achieved, it can be said, it is satisfied or its satisfaction is yes otherwise its satisfaction is no meaning that it has not been completely achieved. The percentage of deviation in achieving a target level means the degree of deviation from that target level. The more there are higher degrees of deviation, the more allocation is unbalanced. To draw a clear conclusion, two above tables are consolidated in the Table 9.

9. Conclusion

In order to compare ICGP & IWGP and select the most appropriate model for balanced allocation of budget among organizational goals, the Table 9 is formed by combining Table 7 and Table 8. As it can be seen from Table 9, both ICGP and IWGP are compared in terms of their solutions.

The following conclusions can be drawn from Table 9:

The first goal is completely achieved (complete achievement) by ICGP Model and completely achieved by IWGP Model. It means that the first goal is completely satisfied by two models. The second goal is completely achieved by ICGP Model and 0.10% underachieved by IWGP Model. It means that the second goal is completely satisfied only by ICGP. The third goal is 13.3% over achieved by ICGP Model and completely achieved by IWGP Model. It means that the third goal is completely satisfied only by IWGP. The fourth goal is completely achieved by ICGP Model and completely achieved by IWGP Model. It means that the fourth goal is completely satisfied by two models. The fifth goal is 50% under achieved by ICGP Model and 100% under achieved (complete deviation) by IWGP Model. It means that none of two models has satisfied this goal. The sixth goal is 75% under achieved by ICGP Model and 100% under achieved by IWGP Model. It means that none of two models has satisfied this goal. The seventh goal is 30% under achieved by ICGP Model and completely achieved by IWGP Model. It means that the goal is completely satisfied only by IWGP. The eighth goal is 25% under achieved by ICGP Model and 100% under achieved by IWGP Model. It means that none of two models has satisfied this goal. The ninth goal is 100% under achieved by ICGP Model and 28% under achieved by IWGP Model. It means that none of two models has satisfied this goal.

In the ICGP, there are only 3 complete achievements whereas there are 4 complete achievements in IWGP. It means that the IWGP Model works better in terms of total satisfaction of target levels but while considering the complete deviations, it becomes clear that the IWGP has a worse performance in terms of complete deviations because it has 3 complete deviations (goal 5, goal 6 and goal 8) while ICGP has only one complete deviation (goal 9). This implies that In IWGP, some goals are completely satisfied at the expense of poor achievements or even complete deviation in other goals (as it can be seen in Table 7). However, though the numbers of complete achievements of ICGP are less than those of IWGP, it is far more balanced than IWGP. The superiority of ICGP in a better balanced allocation of budget can be also proven by looking at the unevenness (disparity) of IWGP results. So, when the balance among the multiple goals is important for planners the ICGP is strongly recommended.

Table 9. Comparison of ICGP and IWGP

Model	Goal 1 Target	Goal 1 Achieved	Degree of unwanted deviation	Model	Goal 1 Target	Goal 1 Achieved	Degree of unwanted deviation
Integer Chebyshev Goal Programming	3000	3000	0%	Integer Weighted Goal Programming	3000	3000	0%
	Goal 2 Target	Goal 2 Achieved			Goal 2 Target	Goal 2 Achieved	
	10000	10000	0%		10000	9000	0.10% under achieved
	Goal 3 Target	Goal 3 Achieved			Goal 3 Target	Goal 3 Achieved	
	45	51	13.3% overachieved		45	45	0%
	Goal 4 Target	Goal 4 Achieved			Goal 4 Target	Goal 4 Achieved	
	10	10	0%		10	10	0%
	Goal 5 Target	Goal 5 Achieved			Goal 5 Target	Goal 5 Achieved	
	6	3	50% under achieved		6	0	100% under achieved
	Goal 6 Target	Goal 6 Achieved			Goal 6 Target	Goal 6 Achieved	
	4	1	75% under achieved		4	0	100% under achieved
	Goal 7 Target	Goal 7 Achieved			Goal 7 Target	Goal 7 Achieved	
	10	7	30% under achieved		10	10	0%
	Goal 8 Target	Goal 8 Achieved			Goal 8 Target	Goal 8 Achieved	
	12	9	25% under achieved		12	0	100% under achieved
Goal 9 Target	Goal 9 Achieved		Goal 9 Target	Goal 9 Achieved			
7	0	100% under achieved	7	5	28% under achieved		

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