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Technical Paper

A robust optimization approach for pollution routing problem with pickup and delivery under uncertainty

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ABSTRACT

Organizations have recently become interested in applying new approaches to reduce fuel consumptions, aiming at decreasing green house gases emission due to their harmful effects on environment and human health; however, the large difference between practical and theoretical experiments grows the concern about significant changes in the transportation environment, including fuel consumptions, carbon dioxide (CO_2) emissions cost and vehicles velocity, that it encourages researchers to design a near-reality and robust pollution routing problem. This paper addresses a new time window pickup-delivery pollution routing problem (TWPDPRP) to deal with uncertain input data for the first time in the literature. For this purpose, a new mixed integer linear programming (MILP) approach is presented under uncertainty by taking green house emissions into consideration. The objective of the model is to minimize not only the travel distance and number of available vehicles along with the capacity and aggregated route duration restrictions but also the amount of fuel consumptions and green house emissions along with their total costs. Moreover, a robust counterpart of the MILP is introduced by applying the recent robust optimization theory. Computational results for several test problems indicate the capability and suitability of the presented MILP model in saving costs and reducing green house gases concurrently for the TWPDPRP problem. Finally, both deterministic and robust mathematical programming are compared and contrasted by a number of nominal and realizations under these test problems to judge the robustness of the solution achieved by the presented robust optimization model.

1. Introduction

Carbon dioxide (CO_2) is one of the elemental green house gases emitted through human resource activity. Increasing CO_2 becomes a major problem for the natural cycle in ecosystem as the nature always maintain the equilibrium between the amount of CO_2 unleashed and the amount of CO_2 refined; therefore, human resource activity is responsible for that rising has come about. CO_2 emissions in United State went up by about 12% between 1990 and 2010 [1]. In 2010, it accounted for quiet 84% of all United State so that the main human resource activity causes CO_2 emissions with the combustion of fossil fuels (e.g., coal, natural gas and oil) for producing energy and transportation. The amount of CO_2 emission of fuel combustion in transportation, such as gasoline and diesel, after electricity producers is the second largest source of CO_2 pollutants. It is responsible for 31% of total United State CO_2 emissions and 26% of total green house gas emissions [1]. The road transportation accounts for 78% of green house emissions, including CO_2 growing the concern about hazardous effects on ecosystem [2], which makes scientists and researchers develop adaptive approaches and models to explicitly reduce the amount of the above pollutants.

The vehicle routing problem (VRP) determines the optimal set of routes to be performed by a fleet of vehicles to serve a given set of customer under specific constraints. Besides, the vehicles are start from, and end to a depot [3–5]. The classical version of the VRP is capacitate VRP (CVRP), in which all customers correspond to deliveries and the demands are deterministic, and it may not be split and only capacity restrictions are considered for the vehicles. Fung et al. [6] represent a capacitate arc routing problem which uses memetic algorithm to find a set of routes holding the minimum cost of transportation. The computational results illustrate that the quality of the solution is high in a reasonable time.

Another common variant is time window VRP (TWVRP) that is an extension of CVRP, at which each customer is associated to an interval, called time window, and each of them must be met at its interval. The significance of the application of this TWVRP in real world has made researchers do efforts to find appropriate and strong algorithm to solve these types of VRP models. For example,

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Nagate et al. [7] introduce a memetic algorithm for TWVRP using new penalty function to eliminate violation of time window constraints as well as original constraint uses in common memetic algorithm. The experimental result of this model asserts that the developed algorithm performs well. Zachariadis et al. [8] represent a hybrid solution approach as a combination of tabu search and guided local search which has ample power to explore a vast space to find a better solution for VRPPD with simultaneous pickup and delivery services. Jeon et al. [9] develop the main VRPPD model and consider double trip, multi depot, and heterogeneous vehicles and suggest a hybrid genetic algorithm in order to solve the large scale problems.

An extended variant in this paper is introduced as VRP with pickup and delivery (VRPPD), in which some customers have demand to deliver and some of others have loads to pickup. The typical class of VRPPD is picking up load from the entire customer related to, and then delivering load to all remained customers. Tersan and Gen [10] consider a pickup delivery problem in which vehicle can deliver and collect goods simultaneously in order to model real world problem such a reverse logistic and then examine the mathematical model by genetic algorithm and represent the results which approved the accuracy of the model. Another introduction of VRPPD with simultaneous pickup delivery announced by Goksal et al. [11]; In this paper, a heuristic solution based on particle swarm optimization is presented which uses annealing-like strategy to preserve the diversity of swarm. For the literature on the VRP and its extensions, readers can be referred to [12–18].

There is a great deal of effort at extending the traditional VRP objectives and constraints not just to account for the economic costs but to consider more comprehensive environmental and ecological impacts. A certain amount of works reduces the CO_2 emissions with lowering the amount of fuel consumptions. By way of illustration, Erdugan and Miller-Hooks [19] introduce a mathematical model by keeping fuel consumptions to a minimum via minimizing the total traveled distance while considering the needs for refueling in the route plans so optimized as to avoid the risk of running out of fuel. Ubeda et al. [20] represent a distance-based model affected by multiplying an emission factor depended on average weight of a typical vehicle and fuel conversion factor. This study shows that introducing backhauls to avoid empty running benefit from both economic and ecological more efficient.

Apart from the above-mentioned studies, some of other works involve factors depended on the features of the vehicle, the fuel that vehicle consumes, and the route it travels to deplete the amount of CO_2 emitted. Suzuki [21] shows that the less distance a delivery vehicle is anticipated with heavier load traveled the less fuel consumptions. Also, they consider the fuel consumptions that are relevant to waiting time in customer service place. Figliozzi [22] represents an analytical model of CO_2 emissions involved in a variety of time-definitive customer demands by using time dependant vehicle routing method in order to plan vehicles routes. Bektas and Laporte [23] shed light on the trade-off between different parameters, such as vehicle load, speed, and aggregate cost and environmental green vehicle routes, and they represent a model named pollution routing problem (PRP).

The review of the related literature indicates that there is a significant gap on the applying the new economic-friendly and ecological-friendly VRP to pickup and delivery systems. Most studies have failed to pay attention to the effect of some economic impacts on ecological influence in the VRP networks. In addition, consuming the realization of some influential parameters seems to be necessary because of the importance of building mathematical models in the real-world applications. This paper is to address a new time window pickup-delivery pollution routing problem (TWPDPRP) to deal with uncertain input data for the first time in the literature. The presented problem combines conventional VRP and technical and mechanical approaches to establish a new mixed integer linear programming (MILP) model by considering economic and environmental issues concurrently.

The main innovations of this paper to differentiate the efforts from those already published on the subject are as follows: (1) introducing penalty costs for pickup nodes in order to create a sequence of pickup nodes to reducing the amount of traveling time and then the amount of fuel consumptions and CO_2 emissions; (2) integrating time window constraints into the VRPPD in order to decrease the arrival time to delivery nodes. Both (1) and (2) show up the economic effects that enrich environmental influences in the model; (3) applying robust optimization within the constraints considering the velocity of vehicle as an uncertain parameter in order to achieve a practical model; for instance, this uncertainty is related to traffic flow, stops at fuel stations and traffic lights. All of the above considerations bring the mathematical model closer to reality, unlike the previous studies by considering the velocity as a set of numbers that can be chosen from in order to optimize the VRP model [22,23]; (4) considering the changes of the CO_2 emissions cost and the fuel consumptions cost over the time to think over the altering in the real world applications. By taking (3) and (4) into consideration robust optimization imports in the TWPDPRP as a new way to create a trade-off between economic and ecological issues and combines them to the natural cases; (5) taking into account the robust concept for service time of each customer to deal with different situations; and (6) thinking over the physical feature of each vehicle, such as the front surface the weight, the slope and the fraction factor of roads and bring them into the TWVRPPD to get the best sequence pickup and delivery nodes so that the fuel consumptions and CO_2 emissions cost get down considerably.

The remainder of this paper is organized as follows. Sections 2, 3 and 4 provide a formal description of the TWPDPRP, the proposed mathematical model and the robust counterpart mathematical model, respectively. Computational results on deterministic and robust solutions are presented in Section 5. Also, in this section the sensitivity analysis is reported on various parameters of the proposed MILP model. The final section ends with the conclusions.

2. Problem definition

This paper addresses a new TWPDPRP including two groups of nodes. The first one contains customers whose loads should be picked up, and the second one covers customers whose demands should be delivered. The model employs a group of vehicles for service tasks, unlike the previous studies with only one vehicle mentioned in Section 1. As illustrated in Fig. 1, after servicing all pickup customers, their loads are carried by trucks (i.e., the loads picked up from the first group nodes and the loads that are needed to be picked up from depot at the beginning for supplying total demands accurately), the vehicles distribute the loads among delivery customers. Hence, the amount of the product which each vehicle loaded up at the start node (i.e., depot) is the total demands of delivery nodes in which they visit during their routes minus the total loads collected from pickup points during traveling the routes that they are associated to. If total picked up products are over or equal to the total delivery demands during the relevant route the vehicle follows, it leaves the depot empty. In addition, the physical feature of each vehicle, such as the front surface the weight, the slope and the fraction factor of roads, are considered based on [24].

Calculating the velocity value: Due to assuming vehicles motion with constant acceleration, the maximum value of velocity during traveling from point i to point j with the distance amount, d_{ij} is achieved from the below equation [24]:

$$v_{ij} = \sqrt{v_0^2 + 2ad_{ij}} \tag{1}$$

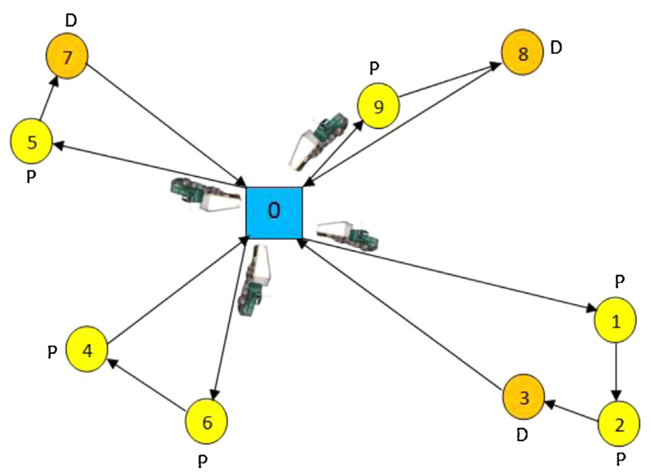

Fig. 1. An illustrative example for the TWPDPRP with nine customers, including six pickup customers (P), one depot (O) and three delivery customers (D).

But, the velocity in constant acceleration motion has different value in various parts of traveling d_{ij}. Thus, the mean value of velocity can be a proper value to figure out the time traveling between points i and j for the deterministic solution calculated by:

$$\bar{v} = \frac{v_{ij} - v_0}{2} \tag{2}$$

v is the maximum velocity and v_0 is the minimum velocity during traveling d_{ij}.

To specify the study scope, assumption and facilitation are supposed in the presented MILP model formulation as below:

- All pickup and delivery nodes must be serviced; however, extra loads are allowed to retrieve to the depot by vehicles at the end of their routes.
- Locations of the depot and customers nodes are fixed and predefined.
- All vehicles end up getting to depot after completing their routes.
- Each vehicle has a predefined and finite capacity.

With considering the above assumptions, this paper sets out to describe a process in order to reduce the average amount of loads carried by each vehicle and to decrease the negative impacts of carrying loads on fuel consumptions. Then, CO_2 emissions beside lowering the travel distance, travel time and arrival times to points for each vehicle by considering the road elements, such as slope and fraction, and the physical features of each vehicle, such as front surface, net weight and capacity. It also links the penalty costs for pickup points to time window intervals of delivery points.

3. Model formulation

The following notations are used in the formulation of the proposed TWPDPRP.

3.1. Sets

P	Set of pickup customers $p = \{1, 2, \ldots, P\}$
D	Set of delivery customers $d = \{n+1, n+2, \ldots, n+n'\}$
K	Set of vehicle $k = \{1, 2, \ldots, K\}$
$\{0\}$	Depot at the beginning of the route (all vehicles starts from)
$\{n+n'+1\}$	Depot at the end of the route (all vehicles end to)
V	Set of all vertices $\{0, 1, 2, \ldots, n, n+1, \ldots, n+n'+1\}$
A	Set of edges $A = \{(i, j) \mid i, j \in V\, j \neq 0, i \neq n+n'+1, i \neq j\}$

3.2. Indices

i	Index of the pickup and delivery customers and the depot at the start and end of the route $i = \{0, 1, \ldots., n, n+1, \ldots, n+n'+n+n'+1\}$
j	Index of pickup and delivery customers plus depot at the start and end of the route $j = \{0, 1, \ldots, n, n+1, \ldots., n+n'+n+n'+1\}$>
k	Index of vehicles $k = \{1, 2, \ldots, K\}$

3.3. Parameters

q_i	Demand for customer i $\begin{cases} q_i > 0 & \text{if } i \in P \\ q_i < 0 & \text{if } i \in D \\ q_0 = q_{n+n'+1} = 0 \end{cases}$
C_k	Capacity of vehicle k
d_i	Service time at vertex i
e_i	Earliest start of service time at vertex i
l_i	Latest service time at vertex i
C_{ijk}	Cost of traveling between vertex i, j by vehicle k
t_{ijk}	Travel time between vertex i, j by vehicle k
T_k	Maximum travel time of vehicle k
D_i	Due date of vertex i
CT_i	Cost of tardiness in vertex i
CE_i	Cost of earliness in vertex i
C_f	Cost of fuel consumption
e	Cost of CO_2 emission
W_k	Weight of vehicle k without loads
P	Cost of driver per hours
θ_{ij}	Slope of edge (i, j)
A_k	Front space of vehicle k
C_d	Friction rate of air
C_r	Rolling resistance
$sin(i, j)$	Sine of the angle between the road connecting vertex i and j and horizon line (angle is calculated based on gradient)
$cos(i, j)$	Cosine of the angle between the road connecting vertex i and j and horizon line (angle is calculated based on gradient)
ρ	Air density
g	Gravity
ϑ_{ijk}	Speed of vehicle k traveling between vertex i and j
a_k	Average acceleration of vehicle k

3.4. Decision variables

X_{ijk}	$\begin{cases} 1 & \text{if vehicle } k \text{ travels between vertex } i \text{ and } j \\ 0 & \text{otherwise,} \end{cases}$
Q_{ijk}	Loads of vehicle k before arriving to j after passing i subsequently
y_{ik}	Time of starting service at vertex i by vehicle k
S_{jk}	Time of completing a route by vehicle k when vertex j is the last node before vertex $n+n'+1$
TT_{ik}	Tardiness time for vehicle k in vertex i
ET_{ik}	Earliness time for vehicle k in vertex i

In terms of the above notations, the proposed MILP model for the TWPDPRP is presented as follows:

$$\begin{aligned} Min\, Z = {} & \sum_{k=1}^{K} \sum_{j=0, i \neq j}^{n+n'+1} \sum_{i=0}^{n+n'+1} (C_f + e)(a_k + (\sin(i, j) \\ & + C_r \cos(i, j))g) d_{ij} W_k x_{ijk} \\ & + \sum_{k=1}^{K} \sum_{j=0, i \neq j}^{n+n'+1} \sum_{i=0}^{n+n'+1} (C_f + e)(a_k + (\sin(i, j) + C_r \cos(i, j))g) d_{ij} Q_{ijk} \\ & + \sum_{k=1}^{K} \sum_{j=0, i \neq j}^{n+n'+1} \sum_{i=0}^{n+n'+1} (C_f + e)(1/2 C_d A_k \rho) d_{ij} t_{ij} x_{ijk} \\ & + \sum_{k=1}^{K} \sum_{i=0}^{n+n'+1} Ps_{ik} + \sum_{k=1}^{K} \sum_{i=0}^{n+n'+1} CE_i ET_{ik} + \sum_{k=1}^{K} \sum_{i=0}^{n+n'+1} CT_i TT_{ik} \end{aligned} \tag{3}$$

s.t.

$$\sum_{j=1}^{n+n'+1} x_{0jk} = 1 \quad \forall k \in K \tag{4}$$

$$\sum_{i=0}^{n+n'} x_{in+n'+1k} = 1 \quad \forall k \in K \tag{5}$$

$$\sum_{k=1}^{K} \sum_{j=1 j \neq i}^{n+n'+1} x_{ijk} = 1 \quad \forall i \in P \cup D \tag{6}$$

$$\sum_{k=1}^{K} \sum_{i=0 j \neq i}^{n+n'} x_{ijk} = 1 \quad \forall j \in P \cup D \tag{7}$$

$$\sum_{k=1}^{K} \sum_{i=0 j \neq i}^{n+n'} x_{ijk} - \sum_{k=1}^{K} \sum_{i=1 j \neq i}^{n+n'+1} x_{jik} = 0 \quad \forall j \in P \cup D \tag{8}$$

$$y_{ik} + t(i,j) + d_i - y_{jk} \leq M(1 - x_{ijk}) \quad \begin{array}{l} \forall i \in P \cup D \cup \{0\}, \\ \forall j \varepsilon P \cup D \cup \left\{ n + n' + 1 \right\}, \\ i \neq j, k \in K \end{array} \tag{9}$$

$$y_{jk} + t(i, n + n' + 1) + d_j - s_{jk} \leq M(1 - x_{in+n'+1k}) \quad j \in P \cup D \cup \left\{ 0 \right\}, k \varepsilon K \tag{10}$$

$$\sum_{k=1}^{K} \sum_{i=0 j \neq i}^{n+n'} Q_{ijk} - \sum_{k=1}^{K} \sum_{i=1 j \neq i}^{n+n'+1} Q_{jik} = -q_j \quad \forall j \in P \cup D \tag{11}$$

$$\max \{0, -q_j\} \leq Q_{ijk} \leq \min \{C_k, C_k - q_j\} \quad i, j \in P \cup D \cup \left\{ 0 \right\}, k \in K \tag{12}$$

$$y_{ik} - y_{jk} \leq M \left(2 - \left(\sum_{l=0, l \neq j}^{n+n'} x_{ljk} - \sum_{l=0, l \neq i}^{n+n'} x_{lik} \right) \right) \quad i \in P, j \in D, k \in K \tag{13}$$

$$\sum_{k=1}^{K} \sum_{j=0}^{n} \sum_{i=n+1}^{n+n'+1} x_{ijk} = 0 \tag{14}$$

$$e_i \sum_{j=0, i \neq j}^{n+n'} x_{ijk} \leq y_{ik} \leq l_i \sum_{j=0, i \neq j}^{n+n'} x_{ijk} \quad \forall k \in K, \forall i \in D \tag{15}$$

$$\max\{0, y_{ik} - D_i\} \leq TT_{ik} + M \left(1 - \sum_{j=0, i \neq j}^{n+n'} x_{jik} \right) \quad \forall i \in P, \forall k \in K \tag{16}$$

$$\max\{0, D_i - y_{ik}\} \leq ET_{ik} + M \left(1 - \sum_{j=0, i \neq j}^{n+n'} x_{jik} \right) \quad \forall i \in P, \forall k \in K \tag{17}$$

$$Q_{0ik} + \sum_{j=1}^{n+n'+1} \sum_{l=0}^{n+n'} q_i x_{ljk} \geq -M(1 - x_{0ik}) \quad \forall i \in V / \left\{ 0 \right\} \tag{18}$$

$$x_{ijk} \in \left\{ 0, 1 \right\} \quad \forall i \in V, \forall j \in V, \forall k \in K \tag{19}$$

$$y_{ik} \geq 0, Q_{ijk} \geq 0, s_{jk} \geq 0, ET_{ik} \geq 0, TT_{ik} \geq 0 \quad \forall i \in V, \forall j \in V, \forall k \in K \tag{20}$$

The objective function (3) minimizes total costs including six components: the first three measure the fuel consumptions cost and CO_2 emissions cost which consider the roads physical condition selected by vehicles to travel (i.e., the road fraction, and road slop), the weight of the vehicles, and the loads they carry, in physical equations that present the amount of energy consumed by their independently and interdependently impacts. Surface and air fraction impacts on fuel consumption are even involved in those equations. Acceleration and velocity impacts on time traveling and vehicle fuel consumptions are considered as well. The sixth component calculates the driver cost on each route. The last two components describe the penalty cost for tardiness and earliness in arrival time of pickup customers. Constraints (4) ensure that all vehicles start their route from the depot. Constraints (5) assure that all vehicles end their routes to the depot. Constraints (6) and (7) consider that each vertex must be visited just one time. Constraints (8) define a flow by which the number of arrivals is equal to the number of departures for all vertices. Constraints (9) are subtours elimination constraints. The total time of traveling for each vehicle is related to the last node that it passes before reaching to the depot. Thus, by constraints (10) the entire traveling time in the corresponding tour calculated by a linear formulation derived from the below nonlinear equation:

$$S_{jk} = \left(y_{jk} t_j + \frac{d_j 0}{v_{j0k}} \right) x_{j0k} \tag{21}$$

The equilibrium of products flow is described by constraints (11). They clear that no load appears or disappears by itself and the loads level each vehicle carries only change by customers order. Constraints (12) are employed to restrict total loads that each vehicle carries by its capacity. Constraints (13) and (14) define restrictions to service all pickup customers before delivery customers. Time window limitation is shown by constraints (15). Constraints (16) and (17) describe the tardiness and earliness of arriving time for each vertex. Constraints (18) disperse the number of loads needed to pickup from the depot at the beginning of the route. Finally, constraints (19) and (20) enforce the binary and non-negativity restrictions on decision variables.

4. Robust counterpart mathematical model

Recall that the linear optimization problem is indicated as follows:

$$\min c^T x \tag{22}$$

$$s.t. Ax \geq b \tag{23}$$

where $c \in R^n$ is vector forming objective function coefficient for decision variables and $x \in R^n$ is the vector of decision variables, and finally $A \in R^{m \times n}$ and $b \in R^m$ are constraint matrix and right hand side vector.

Uncertainty part of the proposed model in this paper is considered to be optimized by the robust counterpart linear optimization solution which is done based on Ben-tal and Nemirofski [25]. Before representing the proposed robust model, some definitions are reviewed for the robust optimization theory.

Definition 1. A feasible solution for robust linear optimization is a set of values related to decision variables that represent a meaning full objective function value besides satisfying all the constraints as represented below:

$$\{\min \{c^T x + d \, Ax \leq b\}\}_{\forall (c,d,A,B) \in u} \tag{24}$$

The difference between (23) and (24) is the data $(c'd, A, b)$ varying in a given detailed closed bounded box u, which is called uncertainty set as well [25], where $u \subseteq R^{(m+1)\times(n+1)}$ and its general form is indicated as follows:

$$\mu = \{\vartheta \in R^n : |\vartheta_t - \hat{\vartheta}_t| \le \rho G_t, t = 1, \ldots, n\}, \quad (25)$$

in which $\hat{\vartheta}_t$ is normal value of ϑ_t, $\rho > 0$ is the level of uncertainty, and G_T defines as uncertainty scale.

Definition 2. By considering a potential solution x, the robust objective function value $\hat{c}(x)$ is the supermom value of that 'true' objective $C^T c + d$ set over whole realizations of the data from u.

$$\hat{c}(x) = \sup\,[c'^T x + d]_{(c,d,A,b)\in} u \quad (26)$$

After knowing the meaningful solutions for uncertain model definition, the uncertain model must suggest the best robust value of objective function among given feasible solutions. This process introduces the robust counterpart of an uncertain optimization problem.

Definition 3. The robust counterpart of an uncertain linear is optimizing the below problem:

$$\min_x\{\hat{c}(x) = \sup\,[c'^T x + d]_{(c,d,A,b)} : Ax \le b\forall(c, d, A, b) \in u\} \quad (27)$$

By solving (27), the robust optimal solution of (26) can be obtained. The value of objective function in Definition 3 can satisfy all constraints and give the best robust optimized value for linear models.

For applying the robust counterpart of the presented model, fuel consumptions cost (C_f), CO_2 emissions cost (e), the service time for each customer serviced by a certain vehicle (d_{ik}), the time traveling between two certain customers (t_{ij}) are assumed as uncertain parameters. Hence, according to above-mentioned definitions, the robust counterpart of the recommended TWPDPRP designs a model for uncertain service time, traveling time, fuel consumptions and CO_2 emissions cost provided by uncertainty set u.

Min Z

s.t.

$$\sum_{k=1}^{K}\sum_{j=0,i\ne j}^{n+n'+1}\sum_{i=0}^{n+n'+1}(\bar{C}_f + \bar{e})(a_k + (\sin(i,j) + C_r\cos(i,j))g)d_{ij}W_k x_{ijk}$$
$$+\sum_{k=1}^{K}\sum_{j=0,i\ne j}^{n+n'+1}\sum_{i=0}^{n+n'+1}(\bar{C}_f + \bar{e})(a_k + (\sin(i,j) + C_r\cos(i,j))g)d_{ij}Q_{ijk}$$
$$+\sum_{k=1}^{K}\sum_{j=0,i\ne j}^{n+n'+1}\sum_{i=0}^{n+n'+1}(C_f + e)(1/2C_dA_k\rho)d_{ij}t_{ij}x_{ijk} + \eta^t_{ij}) + \eta_f + \eta_e$$
$$+\sum_{k=1}^{K}\sum_{i=0}^{n+n'+1}Ps_{ik} + \sum_{k=1}^{K}\sum_{i=0}^{n+n'+1}CE_iET_{ik} + \sum_{k=1}^{K}\sum_{i=0}^{n+n'+1}CT_iTT_{ik} \le Z$$

$$\rho_e G_e x_{ijk} \le \eta_e \quad \forall i, j \in V, \forall k \in K$$

$$\rho_e G_e x_{ijk} \ge -\eta_e \quad \forall i, j \in V, \forall k \in K$$

$$\rho_e G_e Q_{ijk} \le \eta_e \quad \forall i, j \in V, \forall k \in K$$

$$\rho_e G_e Q_{ijk} \ge -\eta_e \quad \forall i, j \in V, \forall k \in K$$

$$\rho_{cf} G_{cf} x_{ijk} \le \eta_{cf} \quad \forall i, j \in V, \forall k \in K$$

$$\rho_{cf} G_{cf} x_{ijk} \ge -\eta_{cf} \quad \forall i, j \in V, \forall k \in K$$

$$\rho_{cf} G_{cf} Q_{ijk} \le \eta_{cf} \quad \forall i, j \in V, \forall k \in K$$

$$\rho_{cf} G_{cf} Q_{ijk} \ge -\eta_{cf} \quad \forall i, j \in V, \forall k \in K$$

$$\rho_t G^t_{ij} x_{ijk} \le \eta^t_{ij} \quad \forall i, j \in V, \forall k \in K$$

$$\rho_t G^t_{ij} x_{ijk} \ge -\eta^t_{ij} \quad \forall i, j \in V, \forall k \in K$$

$$\sum_{j=1}^{n+n'+1} x_{0jk} = 1 \quad \forall k \in K$$

$$\sum_{i=0}^{n+n'} x_{in+n'+1k} = 1 \quad \forall k \in K$$

$$\sum_{k=1}^{K}\sum_{j=1 j\ne i}^{n+n'+1} x_{ijk} = 1 \quad \forall i \in P \cup D$$

$$\sum_{k=1}^{K}\sum_{i=0 j\ne i}^{n+n'} x_{ijk} = 1 \quad \forall j \in P \cup D$$

$$\sum_{k=1}^{K}\sum_{i=0 j\ne i}^{n+n'} x_{ijk} - \sum_{k=1}^{K}\sum_{i=1 j\ne i}^{n+n'+1} x_{jik} = 0 \quad \forall j \in P \cup D$$

$$y_{jk} - y_{ik} \ge -M(1 - x_{ijk}) + t(i, j) + d_i + \rho_d G^d_i + \rho_t G^t_{ij}$$
$$\forall i \in P \cup D \cup \{0\}\ \forall j \in P \cup D \cup \{n + n' + 1\}, i \ne j, k \in K$$

$$s_{jk} - y_{jk} \ge -M(1 - x_{in+n'+1k}) + t\left(i, n + n' + 1\right) + d_j + \rho_d G^d_j$$
$$+ \rho_t G^t_{in+n'+1} \quad j \in P \cup D \cup \{0\}, k \in K$$

$$\sum_{k=1}^{K}\sum_{i=0 j\ne i}^{n+n'} Q_{ijk} - \sum_{k=1}^{K}\sum_{i=1 j\ne i}^{n+n'+1} Q_{jik} = -q_j \quad \forall j \in P \cup D$$

$$\max\{0, -q_j\} \le Q_{ijk} \le \min\{C_k, C_k - q_j\}$$
$$\forall i \in P \cup D \cup \{0\}\ \forall j \in P \cup D \cup i \ne j, k \in K$$

$$y_{ik} - y_{jk} \le M\left(2 - \left(\sum_{l=0,l\ne j}^{n+n'} x_{ljk} - \sum_{l=0,l\ne i}^{n+n'} x_{lik}\right)\right) \quad i \in P, j \in D, k \in K$$

$$\sum_{k=1}^{K}\sum_{j=0}^{n}\sum_{i=n+1}^{n+n'+1} x_{ijk} = 0$$

$$e_i\sum_{j=0,i\ne j}^{n+n'} x_{ijk} \le y_{ik} \le l_i\sum_{j=0,i\ne j}^{n+n'} x_{ijk} \quad \forall k \in K, \forall i \in D$$

$$\max\{0, y_{ik} - D_i\} \leq TT_{ik} + M\left(1 - \sum_{j=0, i \neq j}^{n+n'} x_{jik}\right) \quad \forall i \in P, \forall k \in K$$

$$\max\{0, D_i - y_{ik}\} \leq ET_{ik} + M\left(1 - \sum_{j=0, i \neq j}^{n+n'} x_{jik}\right) \quad \forall i \in P, \forall k \in K$$

$$Q_{0ik} + \sum_{j=1}^{n+n'+1}\sum_{l=0}^{n+n'} q_i x_{ljk} \geq -M(1 - x_{0ik}) \quad \forall i \in V/\{0\}$$

$$x_{ijk} \in \{0, 1\} \quad \forall i \in V, \forall j \in V, \forall k \in K$$

$$y_{ik} \geq 0, Q_{ijk} \geq 0, s_{jk} \geq 0, ET_{ik} \geq 0, TT_{ik} \geq 0 \quad \forall i \in V, \forall j \in V, \forall k \in K$$

5. Experimental results

To show the validity and reliability of the represented model, several numerical experiments are executed and relevant solution results are provided in this section. For nominal experiments, nine test problems with different sizes, as shown in Table 2, are done under nominal data chosen from uniform bounds represented in Table 1 and for each size the experiments are performed by considering six different uncertainty levels (i.e., $\rho = 0.1, 0.3, 0.5, 0.7, 0.9, 1$). Then, five random realizations that have gotten from uniform uncertainty set, [nominal value $-\rho^* G^*$, nominal value $+\rho^* G^*$], are examined for six test problems with (i.e., $\rho = 0.1, 0.5, 1$) to evaluate the performance of the solutions represented by the proposed deterministic and robust models. The second results are given in Table 3.

In the analysis of the random realizations, the mean and standard deviation are used as performance measures for both deterministic and robust models. The uncertainty levels for all parameters in the robust model are the same ($i.e. \rho_e = \rho_{cf} = \rho_t = \rho_d$) and for deterministic models ($\rho = 0$). All results related to the robust and deterministic realizations are given in Table 3.

It is obvious that the result related to the proposed robust optimization model is worse than the ones obtained from the proposed deterministic optimization model. It is because that in the robust optimization all uncertain parameters are regarded as worst cases in practice to lower the risk and lack until it is possible to perform. Moreover, as the uncertainty level is increased the robust objective function degenerates. Hence, by increasing the amount of the uncertainty of the proposed model, the solution has to adapt itself to the uncertain conditions and create worse values. Therefore, increasing the uncertainty level the decision becomes much more sensitive and difficult. This sensitivity and difficulty affects on the robust optimization model to adjust the condition in order to produce the solutions that can reduce the possibility of failure in its work rather than the deterministic model. According to Table 3, the robust model represents the solution with higher values and lower standard deviation rather than the deterministic model.

Table 1
Source of random generations for the nominal data.

Parameters	Corresponding random distribution
e	Uniform (3.15, 4.95)
C_f	Uniform (0.35, 0.55)
d_i	Uniform (0.25, 0.75)
v	Uniform (30, 80)

Table 2
Summary of test results under nominal data.

Problem size $\lvert V\rvert^*\lvert P\rvert^*\lvert D\rvert$	Uncertainty rate (ρ)	Objective function value under nominal data	
		Deterministic	Robust
4*2*2	0.1	77,300.619	77,424.144
	0.3		77,671.194
	0.5		77,918.244
	0.7		78,165.293
	0.9		78,412.343
	1		78,535.868
5*2*3	0.1	114,042.620	114,203.143
	0.3		114,524.187
	0.5		104,845.231
	0.7		115,166.275
	0.9		115,487.319
	1		115,647.841
6*3*3	0.1	129,302.698	129,634.650
	0.3		130,298.553
	0.5		130,962.457
	0.7		131,626.361
	0.9		132,290.265
	1		132,622.217
7*5*2	0.1	147,014.565	147,467.174
	0.3		148,372.393
	0.5		149,277.611
	0.7		150,182.830
	0.9		151,088.048
	1		151,540.658
8*5*3	0.1	192,356.047	193,164.917
	0.3		194,782.658
	0.5		196,400.398
	0.7		198,018.139
	0.9		199,635.880
	1		200,444.750
9*5*4	0.1	197,788.229	209,088.737
	0.3		202,139.392
	0.5		205,040.168
	0.7		207,940.944
	0.9		216,296.487
	1		217,197.456
10*5*5	0.1	208,017.768	209,088.737
	0.3		210,890.674
	0.5		212,692.612
	0.7		214,494.550
	0.9		210,841.719
	1		212,292.107
11*5*6	0.1	265,912.423	267,040.540
	0.3		269,296.773
	0.5		271,553.007
	0.7		273,809.241
	0.9		276,065.474
	1		277,193.591
12*5*7	0.1	277,232.580	278,607.945
	0.3		281,358.676
	0.5		284,109.406
	0.7		286,860.137
	0.9		289,610.868
	1		290,986.233

The sensitivity analysis is conducted on a test problem with ($\rho = 0.3$) and nine customers. It is represented in Tables 4–6. In addition, all mathematical models either deterministic or robust are coded in the optimization software (i.e., GAMS).

In the next sub-section, a further sensitivity analysis is performed for evaluating the capability and effectiveness of the proposed MILP model and the impacts of the parameters used in the model (e.g., CO_2 emissions cost, the capacity of vehicles, and the number of pickup (delivery) centers in a customer zone). All sensitivity analyses are reported based on the selected test problem with nine customers.

Table 3
Summary of test results under realization.

Problem size $\|V\|*\|P\|*\|D\|$	Uncertainty level (ρ)	Mean of objective function values under realizations		Standard deviation of objective function values under nominal data	
		Deterministic	Robust	Deterministic	Robust
6*4*2	0.1	132,689.092	133,010.435	5874.806	5863.886
	0.5	103,816.160	105,052.459	7697.402	7243.613
	1	120,900.173	127,016.307	51,434.472	44,923.780
7*3*4	0.1	147,134.402	147,587.708	4540.966	4523.098
	0.5	158,813.433	160,809.728	22,819.394	22,401.967
	1	187,915.287	194,173.110	37,370.501	35,899.592
8*3*5	0.1	193,851.471	194,679.441	9881.532	9826.449
	0.5	170,134.691	173,692.072	20,079.781	19,419.356
	1	206,627.628	214,373.962	75,911.719	73,291.846
9*6*3	0.1	197,446.889	198,973.708	7047.592	68,531.662
	0.5	202,145.887	206,185.946	48,184.873	46,827.245
	1	230,667.210	239,900.967	72,276.401	68,315.493
10*6*4	0.1	231,087.940	232,096.158	5687.912	5656.778
	0.5	221,868.504	231,621.073	20,200.991	15,319.460
	1	252,548.786	268,822.541	129,005.798	126,364.011
11*5*6	0.1	272,306.884	273,600.724	13,122.635	13,079.201
	0.5	215,498.852	220,558.448	54,576.422	53,566.694
	1	324,444.373	359,772.328	94,246.525	66,031.599

Table 4
Results of sensitivity analysis on number of picks up and deliveries for eight test problems.

Problem size $\|V\|*\|P\|*\|D\|$	Objective function values under nominal data	
	Deterministic	Robust
9*8*1	241,837.029	246,064.209
9*7*2	210,132.374	214,111.610
9*6*3	199,663.385	203,727.521
9*5*4	197,788.229	202,139.392
9*4*5	210,255.321	224,855.175
9*3*6	218,928.635	223,313.315
9*2*7	225,453.320	230,897.073
9*1*8	222,027.539	227,471.292

Table 5
Results of sensitivity analysis on CO_2 emissions cost for deterministic and robust test problem.

Problem size $\|V\|*\|P\|*\|D\|$	CO_2 emission cost	Objective function values under nominal data	
		Deterministic	Robust
9*6*3	1.55	117,630.919	121,590.182
	2.05	130,838.654	134,032.446
	2.55	148,044.837	151,554.886
	3.05	165,251.020	169,073.327
	3.55	182,457.203	186,593.767
	4.05	199,663.385	204,114.207
	4.55	216,869.568	221,634.647
	5.05	234,075.751	239,155.088
	5.55	251,281.934	256,675.528
	6.05	268,488.116	274,195.968
	6.55	285,694.299	291,716.408
	7.05	302,900.482	309,236.848
	7.55	320,106.665	326,757.289
	8.05	337,312.847	347,421.928
	8.55	354,519.030	361,798.169
	9.05	371,725.213	379,318.609
	9.55	388,931.396	396,839.050
	10.05	406,137.578	417,856.874
	10.55	423,343.761	431,879.930

5.1. CO_2 emissions cost

A sensitivity analysis on the CO_2 emissions cost are performed and depicted in Fig. 2. As it can be seen in this figure, when the CO_2 emissions cost increases, the objective function values increase as well.

Table 6
Results of sensitivity analysis on the vehicle capacity for deterministic and robust test problem.

Problem size $\|V\|*\|P\|*\|D\|$	Capacity	Objective function under nominal data	Number of truck used in model
9*6*3	25	Infeasible	Infeasible
	26	Infeasible	Infeasible
	27	Infeasible	Infeasible
	28	199,663.385	4
	29	199,654.385	4
	30	199,645.385	4
	31	199,636.385	4
	32	199,627.385	4
	33	199,618.385	4
	34	199,609.385	4
	35	199,600.384	4
	36	199,591.385	4
	45	199,510.385	4
	47	199,492.380	4
	50	199,465.385	4
	55	183,650.569	4
	57	183,632.569	4
	60	183,605.569	4
	50	212,419.715	3
	55	183,650.567	3
	57	183,632.569	3
	60	183,605.569	3

5.2. Number of pickup (delivery) nodes

According to Figs. 3 and 4, the number of pickup and delivery customers affect on the objective function value. For example, when the customer zone has the least pickup customer, the objective function value stands at the maximum amount the model is able to represent based on the test problem. It occurs because the average amount of the load carried during the route and the time of carrying the loads increase, and so they significantly impact on fuel consumptions; however, as the number of pickup nodes goes up the objective function value decrease as a result of reducing the average amount of loads carried in the routes and the time they carried during those routes. It happens because vehicle gets the choice that loads up the product delivery customers need from the pickup customers during their route instead

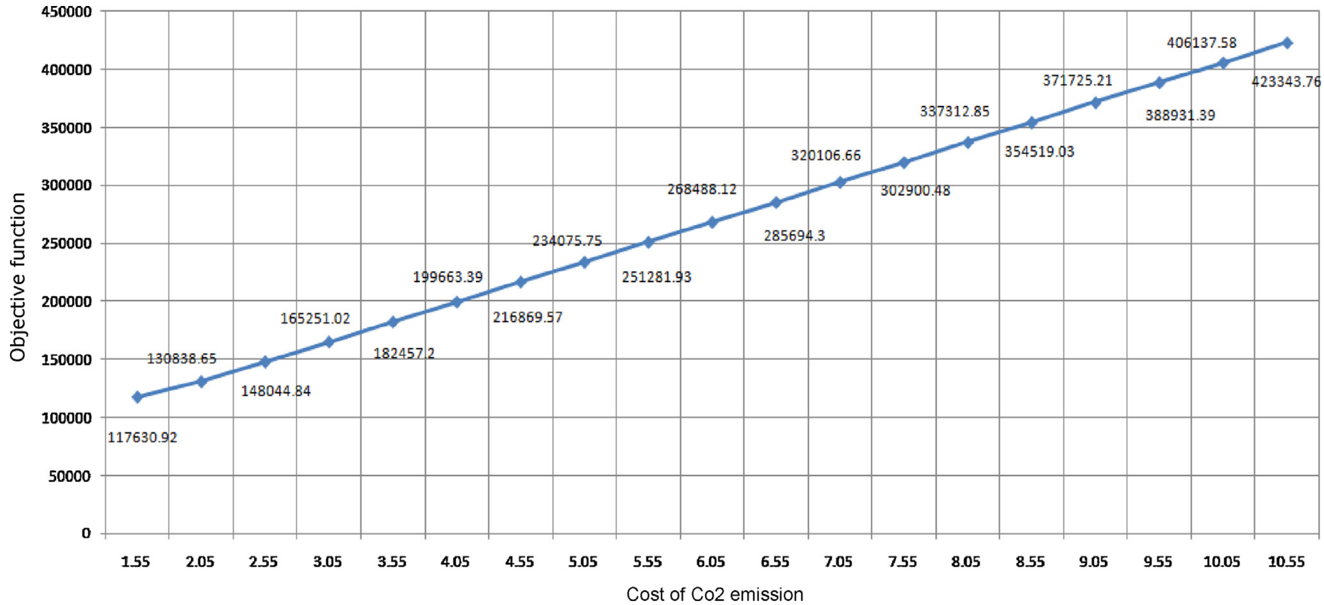

Fig. 2. Objective function and CO_2 emissions cost.

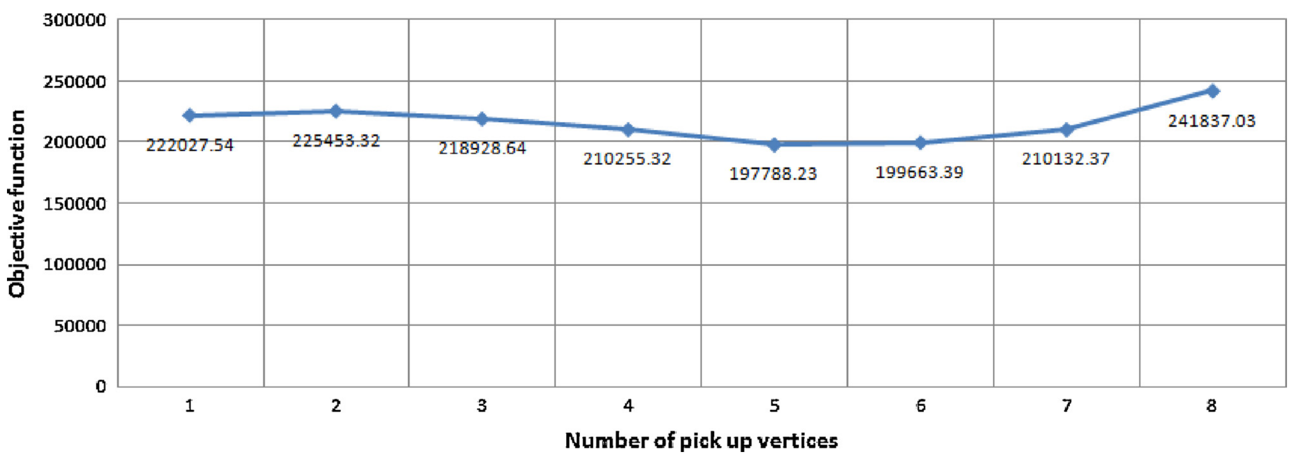

Fig. 3. Objective function and number of pickup nodes.

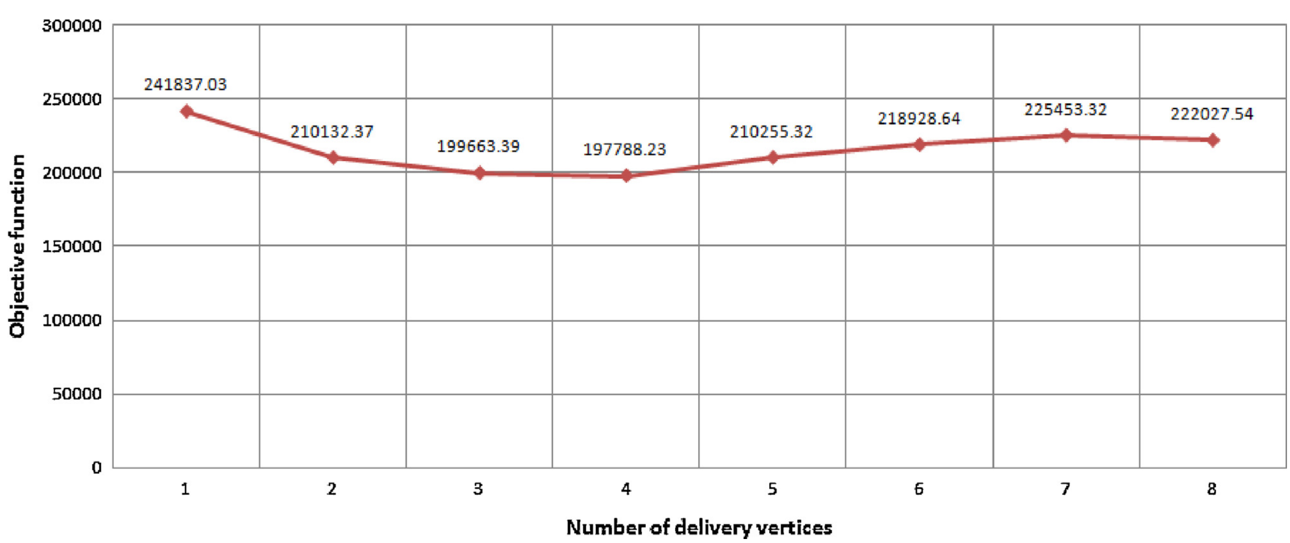

Fig. 4. Objective function and number of delivery nodes.

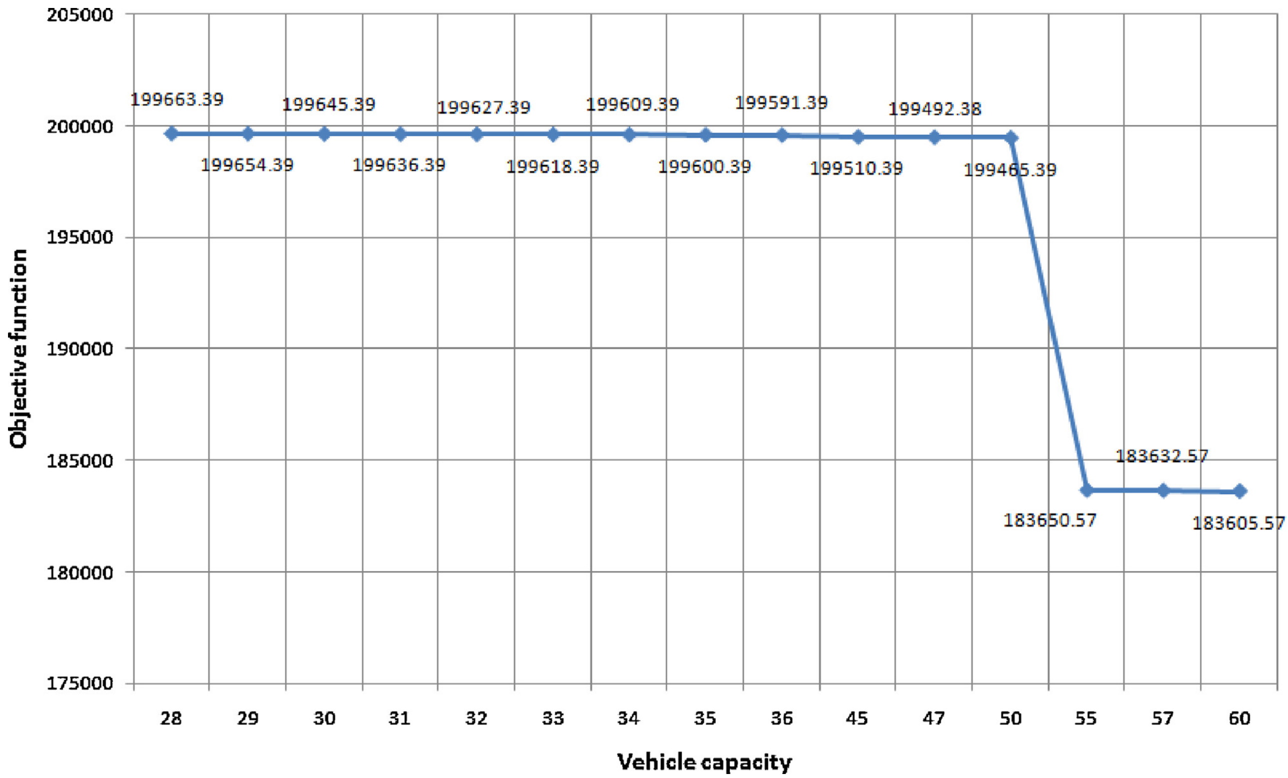

Fig. 5. Objective function and vehicles capacity.

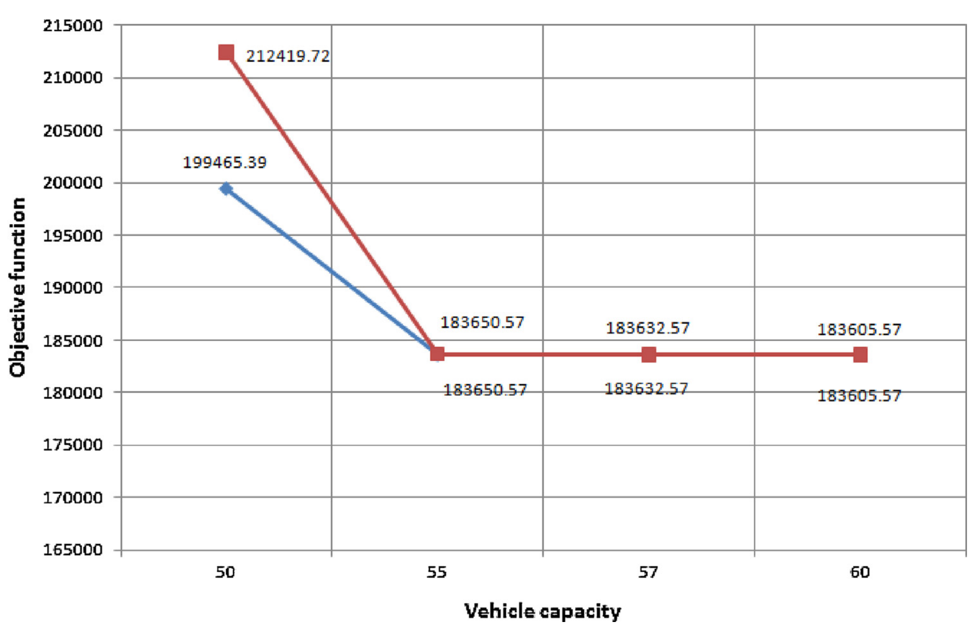

Fig. 6. Objective function and number of vehicles.

of loading up at depot. It can continue until the objective function gets the minimum value and the optimized numbers of pickup and delivery customer is achieved; however, if the number of pickups rises continuously, the objective function value will increase again. It is because of raising the amount of loads which has to be picked up and then carried to the depot, and of course the time of carrying these loads that makes fuel consumed for deliver them to their departures either delivery node or depot.

5.3. Capacity of vehicles

As depicted in Fig. 5, increasing the capacity can not affect the objective function value impressively before a certain size (i.e., 55 tons); however, as it reaches to 55 tons the vehicles becomes capable to change their routes and servicing more customers in a more efficient way. It is obvious that by obtaining to this point for the capacity, the number of vehicles involved in routes decreases. It is pointed out that the number of vehicles and their capacities are interdependent. As it is seen in Fig. 6, the problem can be solved by either three vehicles or four vehicles before reaching to the capacity of 55 tons. In other words, three vehicles are capable to answer all the customers demand; however, they do that in a way consuming more fuel. Thus, four vehicles are more efficient for the capacity up to 55 tons. As the capacity reaches to 55 tons, three vehicles change their routes in the way that objective function values become similar to the objective function value in which four vehicles are used for solving the problem.

6. Conclusion

This paper considers a new time window pickup-delivery pollution routing problem (TWPDPRP) and introduces a new robust mixed integer linear programming (MILP) model under uncertainty based on the recent extensions in the robust optimization theory. A new vehicle routing problem is represented that not only considers the environmental issues but also thinks through the kind of vehicle routing problem, named pickup-delivery vehicle routing problem to reduce the economic and ecologic matters simultaneously. The presented model impacts on pickup arrival times in a way that decreases the tardiness and even earliness of arrival times in order to set them in right time and depend them on the arrival times in delivery customers indirectly which are considered in time windows constraints. On the other hand, the model considers the environmental factors, such as the slop of roads, and friction forces. Then it tries to decline the amount of CO_2 emission and fuel consumption which may increase the cost depended on economic factors. The goal of the model is to strike a balance between economic and environmental factors. In addition, a robust optimization approach is presented for uncertain parameters to model the TWPDPRP, such as fuel consumptions cost, CO_2 emissions cost, travel time and service time. The robustness of the model makes it flexible toward uncertainty and prevents the model from producing infeasible solutions.

The several sensitivity analyses have emphasized that the number of pickup and delivery customers can affect on the objective function value, and an optimized set of them is able to give the best (i.e., minimum) value of objective function among results of other sets. Also, the vehicles capacity impressively takes effects on the customer sets associated to vehicles and the number of vehicles associated to the routes. As a direction for future research, more physical conditions related to roads can be considered in the TWPDPRP problem. Moreover, heterogeneous fleet and their mechanical features, the cost of vehicles set up, and the related analysis of economic costs and benefits for using vehicles can be an important issue to spot.

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