Int J Adv Manuf Technol (2014) 73:687–697
DOI 10.1007/s00170-014-5849-9

ORIGINAL ARTICLE

Robot selection by a multiple criteria complex proportional assessment method under an interval-valued fuzzy environment

Behnam Vahdani · S. Meysam Mousavi ·
R. Tavakkoli-Moghaddam · A. Ghodratnama ·
M. Mohammadi

Received: 12 January 2012 / Accepted: 9 April 2014 / Published online: 4 May 2014

Abstract Recent research is recognizing that multiple criteria analysis should take account of the concepts of uncertainty and risk. In some cases, precise determination of the exact value of alternatives and weights of criteria is difficult. Consequently, to deal with these potential problems, their values are regarded as fuzzy and intervals. This paper proposes an interval-valued fuzzy multiple criteria complex proportional assessment (IVF-COPRAS) method that can reflect both a subjective judgment and objective information in real-life situations. In this method, the performance rating values versus selected criteria as well as the weights of conflicting criteria are linguistic variables represented by interval-valued triangular fuzzy numbers. Moreover, performances of alternatives against subjective criteria are described via linguistic variables and then transformed into interval-valued triangular fuzzy numbers. Finally, an application example for the robot selection problem is given in this paper to show this decision-making steps and the effectiveness of the proposed method in manufacturing companies.

Keywords Robot selection · Multiple criteria analysis · Complex proportional assessment (COPRAS) · Interval-valued fuzzy sets

B. Vahdani (✉)
Faculty of Industrial & Mechanical Engineering, Qazvin Branch, Islamic Azad University, Qazvin, Iran
e-mail: b.vahdani@gmail.com

S. M. Mousavi
Industrial Engineering Department, Faculty of Engineering, Shahed University, Tehran, Iran

R. Tavakkoli-Moghaddam · A. Ghodratnama · M. Mohammadi
School of Industrial Engineering, College of Engineering, University of Tehran, Tehran, Iran

1 Introduction

In the last decade, robots have been utilized in various advanced manufacturing systems for a wide range of applications. An industrial robot is regarded as a reprogrammable multi-functional manipulator, designed to move materials, parts, tools, or other devices by means of variable programmed motions, and to conduct different other tasks [1]. When a manufacturing company makes a decision to purchase a robot to perform a material-handling task, the evaluation and selection have to be conducted among different models and types by considering numerous conflicting specifications, including man–machine interface, programming flexibility, vendor's service contract, load capacity, positioning accuracy, purchase cost, etc.

In the literature, there are some studies that presented models to solve robot selection problems. For instance, Knott and Getto [2] designed a model to assess various robotic systems in an uncertain environment, and potential alternatives were assessed by calculating the total net present values of cash flows of investment, labor components, and overheads. Agrawal et al. [3] applied the technique for order preference by similarity to an ideal solution (TOPSIS) method for the robot selection, but the subjective criteria were not considered in this study. Liang and Wang [4] presented a robot selection algorithm by integrating the concepts of fuzzy set theory and hierarchical structure analysis. The algorithm was utilized to aggregate decision makers' fuzzy evaluations for the criteria weightings and to provide fuzzy indices. Goh [5] used the analytic hierarchy process (AHP) method for the robot selection.

Parkan and Wu [6] proposed decision-making and performance measurement models with industrial applications to the robot selection. Khouja and Kumar [7] applied options theory and an investment assessment approach for selection of robots. Braglia and Petroni [8] performed an investment assessment by using the data envelopment analysis (DEA) model for the robot selection. Layek and Resare [9] designed a decision support system (DSS) according to analytical algorithms in order to choose machining centers and robots simultaneously. Chu and Lin [10] discussed the limitations of the Liang and Wang method [4]. They presented a TOPSIS method for robot selection in a fuzzy environment. Bhangale et al. [11] stated a number of robot selection criteria and prioritized the robots by using TOPSIS and graphical methods. Karsak and Ahiska [12] presented a practical common-weight decision-making approach using the DEA method by considering an improved discriminating power for the technology selection. Rao and Padmanabhan [13] proposed an approach based on digraph and matrix methods for the assessment of industrial robots. A robot selection index was introduced that prioritized robots for an industrial application. The digraph was also presented in terms of the criteria of robot selection and their relative importance. Chatterjee et al. [14] utilized two compromise ranking (VlseKriterijumska Optimizacija I Kompromisno Resenje, VIKOR) and outranking (elimination and choice translating reality, ELECTRE) methods. They compared their relative performance for an industrial application.

In the fuzzy sets theory to cope with uncertainty, it is often difficult for an expert or decision maker (DM) to exactly quantify his or her opinion as a number in interval [0, 1]. Therefore, it is more suitable to represent this degree of certainty by an interval. Sambuc [15] and Grattan [16] noted that the presentation of a linguistic expression in the form of fuzzy sets is not enough properly. Interval-valued fuzzy sets were suggested for the first time by Gorzlczany [17] and Turksen [18]. Also, Cornelis et al. [19] and Karnik and Mendel [20] noted that the main reason for proposing this new concept is the fact that in the linguistic modeling of a phenomenon, the presentation of the linguistic expression in the form of ordinary fuzzy sets is not clear enough. Wang and Li [21] defined interval-valued fuzzy numbers and gave their extended operations. Interval-valued fuzzy sets have been widely used in real-world applications, for instance, Sambuc [15] in thyrodian pathology, Gorzlczany [17] and Bustine [22] in approximate reasoning, and Turksen [18, 23, 24] in interval-valued logic and in preference modeling. Mustajoki et al. [25] utilized intervals in the simple multi-attribute rating technique (SMART) and the weighted methods. Halouani et al. [26] proposed two new multi-criteria 2-tuple group decision methods called preference ranking organization method for enrichment evaluation multi-decision maker 2-tuple-I and II (PROMETHEE-MD-2T-I and II). They integrated their procedure with both quantitative and qualitative information in an uncertain context. Vahdani et al. [27–29] extended VlseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR), the elimination and choice translating reality (ELECTRE), and TOPSIS methods based on interval-valued fuzzy set for solving multiple criteria decision making (MCDM) problems. Yao and Yu [30] utilized statistical data to derive level interval-valued fuzzy numbers to represent unknown alternative effectiveness scores.

The review of the literature indicates that the presented models generally try to solve either the elimination phase which provides a feasible set of alternative robots or the ranking phase of the robot selection problem. The literature survey illustrates that there is a need for a decision-making method in terms of multiple conflicting criteria that integrates the concepts of interval-valued fuzzy sets and compromise programming for the selection problems to cope with uncertainty and risk issues, and to fulfill the needs of the DMs. In this paper, the proposed interval-valued fuzzy multiple criteria complex proportional assessment (IVF-COPRAS) method aims to fill this gap in the robot selection problem. The interval-valued fuzzy numbers are utilized because of the linguistic modeling of a phenomenon, where the presentation of the linguistic expression in the form of ordinary fuzzy sets is not clear enough [27–29, 31, 32].

On the other hand, the COPRAS method under uncertainty is taken into consideration in this paper among the well-known multiple criteria analysis methods, such as AHP, VIKOR, and TOPSIS. Regarding the AHP, in spite of its popularity by increasing the number of criteria as well as alternatives, the method requires a considerable amount of time to complete the pair-wise comparison processes [33, 34]. Thus, it becomes impractical due to lengthy calculations. The VIKOR method is sensitive to "v" value which stands for weighting reference [35, 36]. It is difficult to find an exact value for the weighting reference in the robot selection problem. Also, the TOPSIS method is not efficient enough because it develops two reference points; however, this method does not consider the relative importance of the distances from these points [37, 38]. Furthermore, the conventional COPRAS method has been widely used in a variety of decision-making problems, for instance, the fields of construction management, property management, and economics, with appropriate results in the recent years [39, 40].

The proposed IVF-COPRAS method assumes direct and proportional dependence of the weight and utility degree of investigated versions on a system of criteria adequately describing the alternatives and on values as well as weights of the selected criteria. The proposed method contains a stepwise evaluating procedure of alternatives in terms of significance and utility degree in an interval-valued fuzzy environment. The set of criteria is determined, and experts or DMs compute their values and initial weights under uncertainty. The interested experts or DMs by considering their goals and the existing capabilities can investigate and correct all the information. In canonical MCDM or multiple criteria analysis methods as well as canonical COPRAS method, the performance ratings and the weights of the conflicting criteria are determined precisely. However, under many conditions, crisp data are inadequate to model real-life situations since human judgments including preferences are often vague and cannot estimate his/her preference with an exact numerical value. A more realistic and practical approach may be to utilize linguistic assessments instead of numerical values, that is, to suppose that the ratings and weights of the criteria in the problem are described by means of linguistic variables. Hence, this paper presents an interval-valued fuzzy complex proportional assessment approach which can reflect both subjective judgment and objective information in real-life situations. In this method, the performance rating values as well as the weights of criteria are linguistic variables expressed as interval-valued triangular fuzzy numbers. Furthermore, this paper appraises the performance of potential alternatives against subjective criteria via linguistic variables represented as interval-valued triangular fuzzy numbers. Finally, for the purpose of illustrating the applicability and suitability of the proposed method, an application example from the literature is presented for the robot selection problem.

The remaining of this paper is organized as follows: In Section 2, we briefly introduce the original COPRAS method. Section 3 develops COPRAS method under an interval-valued fuzzy environment to solve selection problems. Section 4 investigates an illustrative example including an application to select a robot among potential candidates. Discussion of results is provided in Section 5. The paper is concluded in Section 6.

2 Multiple criteria complex proportional assessment (COPRAS) method

The MCDM or multiple criteria analysis provides an effective framework for comparison based on the evaluation of multiple conflicting criteria. The MCDM is one of the fastest growing areas of operational research, as it is often realized that many concrete problems can be represented by several criteria. It was described as the most well-known branch of decision-making to solve selection problems [41–45]. The decision process of selecting an appropriate alternative usually has to take many factors into considerations, for instance, organizational needs and goals, risks, benefits, limited resources, etc. Several qualitative and quantitative criteria may affect mutually when evaluating alternatives, which may make the selection process complex and challenging. The method of multiple criteria complex proportional assessment (COPRAS) as one of the well-known MCDM method was first introduced by Zavadskas and Kaklauskas [46]. The method regards the performance rating of each alternative with respect to several criteria and the corresponding criteria weights. It chooses the best alternative by considering both the positive-ideal and the negative-ideal solutions. This well-known method has been applied to solve a wide range of decision problems with remarkable results in the related literature [39, 40]. It is a comprehensive evaluation approach that tries to rank alternatives, described in terms of different criteria. The procedure of the canonical method of complex proportional evaluation consists of the following steps [39, 46, 47]:

Step 1 Selecting the available set most important criteria which describe alternatives.

Step 2 Preparing the decision-making matrix (X) for a MCDM problem in which $A_1, A_2, \ldots, A_m$ are m possible alternatives and $C_1, C_2, \ldots, C_n$ are n criteria.

$$X = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix} \quad i = 1, 2, \ldots, m; j = 1, 2, \ldots, n. \tag{1}$$

Step 3 Determining weights of the criteria q_j.

Step 4 Normalizing the decision-making matrix $\overline{X}$. The normalized values of this matrix are calculated as follows:

$$\overline{x}_{ij} = \frac{x_{ij}}{\sum_{i=1}^{m} x_{ij}} \qquad i = 1, 2, \ldots, m; j = 1, 2, \ldots, n. \tag{2}$$

After this step, we have normalized decision-making matrix as follows:

$$\overline{X} = \begin{bmatrix} \overline{x}_{11} & \overline{x}_{12} & \cdots & \overline{x}_{1n} \\ \overline{x}_{21} & \overline{x}_{22} & \cdots & \overline{x}_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \overline{x}_{m1} & \overline{x}_{m2} & \cdots & \overline{x}_{mn} \end{bmatrix} \quad i = 1, 2, \ldots, m; j = 1, 2, \ldots, n. \tag{3}$$

Step 5 Calculating the weighted normalized decision matrix $\widehat{X}$. The weighted normalized values $\widehat{x}_{ij}$ are calculated by the following:

$$\widehat{x}_{ij} = \overline{x}_{ij} \cdot q_j \quad i = 1, 2, \ldots, m; j = 1, 2, \ldots, n. \tag{4}$$

After this step, we have weighted normalized decision-making matrix as follows:

$$\hat{X} = \begin{bmatrix} \hat{x}_{11} & \hat{x}_{12} & \cdots & \hat{x}_{1n} \\ \hat{x}_{21} & \hat{x}_{22} & \cdots & \hat{x}_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \hat{x}_{m1} & \hat{x}_{m2} & \cdots & \hat{x}_{mn} \end{bmatrix} \quad i = 1, 2, \ldots, m; j = 1, 2, \ldots, n. \tag{5}$$

Step 6 Calculating sums P_i of criteria values which larger values are more preferable for each alternative as follows:

$$P_i = \sum_{j=1}^{k} \widehat{x}_{ij}. \tag{6}$$

In Eq. (6), k is number of benefit criteria. It is assumed that in the decision-making matrix, columns first of all are placed benefit criteria and ones which cost criteria are placed after.

Step 7 Calculating sums R_i of criteria values which smaller values are more preferable for each alternative as follows:

$$R_i = \sum_{j=k+1}^{n} \widehat{x}_{ij}. \tag{7}$$

Step 8 Determining the minimum value of R_i.

$$R_{\min} = \min_i R_i \quad i = 1, 2 \ldots, m \tag{8}$$

Step 9 Calculating the relative weight of each alternative Q_i by the following:

$$Q_i = P_i + \frac{R_{\min} \sum_{i=1}^{m} R_i}{R_i \sum_{i=1}^{m} \frac{R_{\min}}{R_i}}. \tag{9}$$

Equation (8) can be written as follows:

$$Q_i = P_i + \frac{\sum_{i=1}^{m} R_i}{R_i \sum_{i=1}^{m} \frac{1}{R_i}}. \tag{10}$$

Step 10 Determining the priority of the alternative. The greater significance (relative weight of alternative) Q_i, the higher is the priority of the alternative. In the case of $Q_{\max}$, the satisfaction degree is the highest.

Step 11 Calculating the utility degree of each alternative:

$$N_j = \frac{Q_j}{Q_{\max}} 100\%, \tag{11}$$

where Q_j and $Q_{\max}$ are the significance of alternatives obtained from Eq. (9).

3 Proposed interval-valued fuzzy COPRAS (IVF-COPRAS) method

In fuzzy MCDM problems, performance rating values and relative weights are often characterized by fuzzy numbers. A fuzzy number is a convex fuzzy set, defined by a given interval of real numbers, each with a membership value between 0 and 1. Considering the fact that, in some cases, determining precisely this value is difficult, the membership value can be expressed as an interval, consisting real numbers. In this paper, performance rating values as well as criteria weights are regarded as linguistic variables. The concept of linguistic variable is very useful in dealing with situations that are too complex or ill-defined to be reasonably described in conventional quantitative expressions [48]. These linguistic variables can be converted to interval-valued triangular fuzzy numbers as provided in Tables 1 and 2.

Let $\widetilde{X} = [\widetilde{x}_{ij}]_{m \times n}$ be a fuzzy decision matrix for a MCDM problem in which $A_1, A_2, \ldots, A_m$ are m possible alternatives and $C_1, C_2, \ldots, C_n$ are n criteria. So, the performance of alternative

Table 1 Definitions of linguistic variables for the ratings

Linguistic variables	Interval-valued triangular fuzzy numbers
Very poor (VP)	[(0,0);0;(1,1.5)]
Poor (P)	[(0,0.5);1;(2.5,3.5)]
Moderately poor (MP)	[(0,1.5);3;(4.5,5.5)]
Fair (F)	[(2.5,3.5);5;(6.5,7.5)]
Moderately good (MG)	[(4.5,5.5);7;(8,9.5)]
Good (G)	[(5.5,7.5);9;(9.5,10)]
Very good (VG)	[(8.5,9.5);10;(10,10)]

Table 2 Definitions of linguistic variables for the importance of each criterion

Linguistic variables	Interval-valued triangular fuzzy numbers
Very low (VL)	[(0,0);0;(0.1,0.15)]
Low (L)	[(0,0.05);0.1;(0.25,0.35)]
Medium low (ML)	[(0,0.15);0.3;(0.45,0.55)]
Medium (M)	[(0.25,0.35);0.5;(0.65,0.75)]
Medium high (MH)	[(0.45,0.55);0.7;(0.8,0.95)]
High (H)	[(0.55,0.75);0.9;(0.95,1)]
Very high (VH)	[(0.85,0.95);1;(1,1)]

A_i with respect to criterion C_j is denoted as $\widetilde{x}_{ij}$. $\widetilde{x}_{ij}$ and $\widetilde{w}_j$ are expressed in interval-valued triangular fuzzy numbers. It is worth noting that the use of interval-valued numbers gives an opportunity to experts or DMs to define lower and upper bounds values as an interval for the decision matrix's elements and weights of criteria. Also, in a group decision-making environment with K persons, the importance of the criteria and the performance rating of alternatives versus each criterion can be calculated by the following:

$$\widetilde{x}_{ij} = \frac{1}{K}\left[\widetilde{x}_{ij}^{1} + \widetilde{x}_{ij}^{2} + \ldots + \widetilde{x}_{ij}^{k}\right] \tag{12}$$

$$\widetilde{w}_{ij} = \frac{1}{K}\left[\widetilde{w}_{ij}^{1} + \widetilde{w}_{ij}^{2} + \ldots + \widetilde{w}_{ij}^{k}\right] \tag{13}$$

Equations (12) and (13) represent the average values of $\widetilde{x}_{ij}$ and $\widetilde{w}_{ij}$ denoted by experts, where (+) is the sum operator and is applied to the interval-valued fuzzy numbers as defined in Definition A.1 in Appendix. The output is also an interval-valued fuzzy number. The concept of the proposed IVF-COPRAS method is based on the significance and utility degree of each alternative in the multiple criteria analysis under uncertainty. Generally, the proposed method consists of four main steps:

- First, calculation of each interval-valued fuzzy weighted normalized decision matrix;
- Second, calculation of sums of maximizing interval-valued fuzzy indexes and minimizing interval-valued fuzzy indexes representing each alternative;
- Third, determination of interval-valued fuzzy significance of each alternative based on positive and negative alternative characteristics; and
- Fourth, calculation of interval-valued fuzzy utility degree of each alternative.

Now, the proposed approach to develop the COPRAS method under an interval-valued fuzzy environment can be presented as follows:

Step 1 Given $\widetilde{x}_{ij} = \left[(x_{1ij}, x_{2ij}); x_{3ij}; (x_{4ij}, x_{5ij})\right]$, the normalized performance rating can be calculated by:

$$\widetilde{n}_{ij} = \left[\left(\frac{x_{1ij}}{x_{5j}^{+}}, \frac{x_{2ij}}{x_{5j}^{+}}\right); \frac{x_{3ij}}{x_{5j}^{+}}; \left(\frac{x_{4ij}}{x_{5j}^{+}}, \frac{x_{5ij}}{x_{5j}^{+}}\right)\right], \quad i = 1, \ldots, m, \quad j \in \Omega_b \tag{14}$$

$$\widetilde{n}_{ij} = \left[\left(\frac{x_{1j}^{-}}{x_{5ij}}, \frac{x_{1j}^{-}}{x_{4ij}}\right); \frac{x_{1j}^{-}}{x_{3ij}}; \left(\frac{x_{1j}^{-}}{x_{2ij}}, \frac{x_{1j}^{-}}{x_{1ij}}\right)\right], \quad i = 1, \ldots, m, \quad j \in \Omega_c \tag{15}$$

where

$$x_{5j}^{+} = \operatorname*{Max}_{i} x_{5ij}, \quad j \in \Omega_b$$

$$x_{1j}^{-} = \operatorname*{Min}_{i} x_{1ij}, \quad j \in \Omega_c$$

where Ω_b is associated with benefit criteria, and Ω_c is associated with cost criteria. Hence, the normalized matrix $\widetilde{N} = \left[\widetilde{n}_{ij}\right]_{n \times m}$ can be obtained. The above-mentioned normalization method is to preserve the property that the ranges of normalized interval numbers fall within $[0,1]$.

Step 2 Determining the weighted normalized matrix.

Normalization transforms performance rating matrix measured with different units, such as points, ratio, and percentage, into weighted dimensionless variables, allowing their direct comparison. The purpose of this step is to receive dimensionless weighted values of the decision matrix from the comparative indexes. By considering the different importance of each criterion, we can construct the weighted normalized fuzzy decision matrix as follows:

$$\widetilde{V} = \left[\widetilde{v}_{ij}\right]_{n \times m} \tag{16}$$

where

$$\widetilde{v}_{ij} = \widetilde{w}_j \times \widetilde{n}_{ij}. \tag{17}$$

According to Definition A.1 in Appendix, the multiply operator can be applied as follows:

$$\widetilde{v}_{ij} = \left[\left(w_{1j} \times n_{1ij}, w_{2j} \times n_{2ij}\right); w_{3j} \times n_{3ij}; \left(w_{4j} \times n_{4ij}; w_{5j} \times n_{5ij}\right)\right]. \quad (18)$$

Step 3 Calculating sums $\widetilde{P}_i$ of criteria values. The larger for the positive values (maximizing values) is more preferable and better satisfied for each alternative. Sums are calculated according to the following formula:

$$\widetilde{P}_i = \sum_{j=1}^{k} \widetilde{v}_{ij} = \sum_{j=1}^{k} \left[\left(v_{1ij}, v_{2ij}\right); v_{3ij}; \left(v_{4ij}, v_{5ij}\right)\right]. \quad (19)$$

In Eq. (6), k is number of benefit criteria. In fact, the value of weight of the selected benefit criterion can be proportionally distributed among all potential alternative versions and can be aggregated according to their values $\widetilde{v}_{ij}$.

Step 4 Calculating sums $\widetilde{R}_i$ of criteria values. The smaller for the negative values (minimizing values) is more preferable for each alternative. Sums are calculated according to the following formula:

$$\widetilde{R}_i = \sum_{j=k+1}^{n} \widetilde{v}_{ij} = \sum_{j=k+1}^{n} \left[\left(v_{1ij}, v_{2ij}\right); v_{3ij}; \left(v_{4ij}, v_{5ij}\right)\right]. \quad (20)$$

In fact, the value of weight of the selected cost criterion can be proportionally distributed among all potential alternative versions and can be aggregated according to their values $\widetilde{v}_{ij}$.

Step 5 Determining the minimum value of $\widetilde{R}_i$ with respect to Definition A.2 in Appendix.

$$\widetilde{R}_{\min} = \min_i \widetilde{R}_i \quad i = 1, 2 \ldots, m \quad (21)$$

Step 6 Calculating interval-valued fuzzy relative weight of each alternative $\widetilde{Q}_i$ as follows:

$$\tilde{Q}_i = \tilde{P}_i + \frac{\sum_{i=1}^{m} \tilde{R}_i}{\tilde{R}_i \sum_{i=1}^{m} \frac{1}{\tilde{R}_i}} = [(P_{1i}, P_{2i}); P_{3i}; (P_{4i}, P_{5i})] + \frac{\sum_{i=1}^{m} [(R_{1i}, R_{2i}); R_{3i}; (R_{4i}, R_{5i})]}{[(R_{1i}, R_{2i}); R_{3i}; (R_{4i}, R_{5i})] \sum_{i=1}^{m} \frac{1}{[(R_{1i}, R_{2i}); R_{3i}; (R_{4i}, R_{5i})]}} = [(Q_{1i}, Q_{2i}); Q_{3i}; (Q_{4i}, Q_{5i})] \quad (22)$$

The first term of $\widetilde{Q}_i$ increases for higher positive criteria $\widetilde{P}_i$, while the second term of $\widetilde{Q}_i$ increases with lower negative criteria $\widetilde{R}_i$. The significance (priority) of each alternative can be determined on the basis of describing positive and negative criteria that characterize the potential alternative in the multi-criteria analysis. Hence, a higher value of $\widetilde{Q}_i$ can correspond to a more suitable alternative.

Step 7 Determining the priority of the alternative. The greater significance (interval-valued fuzzy relative weight of alternative) $\widetilde{Q}_i$ with respect to Definition A.2 in Appendix, the higher is the priority of the alternative. In the case of $\widetilde{Q}_{\max}$, the satisfaction degree is the highest.

The analysis of this step makes it possible to denote that it may be easily applied to evaluate potential alternatives and choose the most efficient of them while being fully aware of the physical meaning of the decision-making process. Furthermore, it provides a reduced criterion $\widetilde{Q}_i$ formulated under an interval-valued fuzzy environment which is directly proportional to the relative effect of the compared criteria values $\widetilde{v}_{ij}$ and their weights on the computational results. In fact, significance $\widetilde{Q}_i$ of each alternative states the degree of satisfaction of demands and aims pursued by the interested experts or DMs. It means that the greater the $\widetilde{Q}_i$ is, the higher efficiency of the potential alternative.

Step 8 Calculating the utility degree of each alternative.

$$N_j = \frac{h\left(\widetilde{Q}_j\right)}{\left(\widetilde{Q}_{\max}\right)} 100\% \quad (23)$$

where $h\left(\widetilde{Q}_j\right)$ and $h\left(\widetilde{Q}_{\max}\right)$ are the significance of alternatives obtained from Eq. (22) and Definition A.2 in Appendix.

The final step is to determine the best alternative that satisfies different criteria in the multiple criteria analysis. By increasing or decreasing the priority of the potential alternative, its degree of utility also increases or decreases. The degree of the utility is determined by comparing each potential alternative with the most efficient one. The best alternative (candidate) that satisfies several conflicting criteria presented by the highest degree of utility N_j equaling 100 %. All utility values associated with the potential alternatives can range from 0 to 100 %, between the worst and best alternative out of those under consideration. This step can properly facilitate a visual evaluation of the potential alternative's efficiency.

Table 3 Robot selection criteria

Subjective criteria	Objective criteria
Man–machine interface (C_1)	Load capacity (C_4)
Programming flexibility (C_2)	Positioning accuracy (C_5)
Vendor's service contract (C_3)	Purchase cost (C_6)

4 Application of the proposed IVF-COPRAS method in solving the robot selection problem

The illustrative example from Liang and Wang [4] is used to show the feasibility and suitability of the proposed IVF-COPRAS method. Assume that a manufacturing company requires a robot to perform a material-handling task. Three robots, R_1,R_2, and R_3, are chosen for further evaluation.

A committee of four decision makers, D_1,D_2,D_3, and D_4, is formed to conduct the evaluation and to select the most suitable robot. The robot selection criteria and the importance weights of the criteria are shown in Tables 3 and 4, respectively. The ratings of three subjective criteria are shown in Table 5. The data of objective criteria is shown in Table 6.

The normalized decision-making matrix and the weighted normalized decision-making matrix are calculated. The respective results have been presented in Tables 7 and 8 (step 1).

Sums $\widetilde{P}_i$, $\widetilde{R}_i$, and minimum value of $\widetilde{R}_i$ are calculated using Eqs. (19 and 20) (steps 2 to 4). Then, interval-valued fuzzy relative weight $\widetilde{Q}_i$ and utility degree N_i are calculated for each alternative using Eqs. (22 and 23). The results have been presented in Table 9 (steps 5 to 7).

According to Table 9, the ranking order of the three robots is R_3, R_2, and R_1. Hence, the best selection is robot 3.

5 Discussion

In this section, we utilize the interval-valued fuzzy TOPSIS method presented by Ashtiani et al. [31] for the purpose of ranking the robots (potential alternatives) and comparing the results in the robot selection problem. By virtue of evaluation

Table 4 Weights of criteria and the average weights

Criteria	Decision makers				Average weights
	D_1	D_2	D_3	D_4	
C_1	H	VH	VH	H	[(0.7,0.85);0.95;(0.975,1)]
C_2	VH	H	VH	M	[(0.625,0.75);0.85;(0.9,0.9375)]
C_3	M	L	M	L	[(0.125,0.2);0.3;(0.45,0.55)]
C_4	VH	VH	H	VH	[(0.775,0.9);0.975;(0.9875,1)]
C_5	VH	H	H	H	[(0.625,0.8);0.925;(0.9625,1)]
C_6	M	M	M	L	[(0.1875,0.275);0.4;(0.55,0.65)]

Table 5 Ratings of robots under subjective criteria and the average ratings

Criteria	Robots	Decision makers				Average ratings
		D_1	D_2	D_3	D_4	
C_1	R_1	F	F	G	VG	[(4.75,6);7.25;(8.125,8.75)]
	R_2	F	G	F	F	[(3.25,4.5);6;(7.25,8.125)]
	R_3	G	F	VG	G	[(5.5,7);8.25;(8.875,9.375)]
C_2	R_1	G	P	G	F	[(3.375,4.75);6;(7,7.75)]
	R_2	VG	G	VG	F	[(6.25,7.5);8.5;(9,9.375)]
	R_3	G	F	VG	G	[(5.5,7);8.25;(8.875,9.375)]
C_3	R_1	F	F	G	F	[(3.25,4.5);6;(7.25,8.125)]
	R_2	G	F	VG	G	[(5.5,7);8.25;(8.875,9.375)]
	R_3	G	G	G	VG	[(6.25,8);9.25;(9.625,10)]

of criteria with respect to each other and the evaluation of alternatives with respect to criteria, the computational results are provided in Table 10 for the robot selection problem. By considering Tables 9 and 10, it is observed that results of the proposed IVF-COPRAS method and the recent interval-valued fuzzy TOPSIS (IVF-TOPSIS) method by Ashtiani et al. [31] are similar. Both interval-valued fuzzy decision-making methods provide similar priorities for the robot selection problem, in which the first rank (R_3) is the same. Both decision methods take low calculation time, particularly when we utilize a spreadsheet software. They are based on the utility theory and can solve the real-life decision-making problems involving any number of qualitative and quantitative criteria as well as any number of alternatives.

In sum, the main contributions and benefits of the proposed IVF-COPRAS method in comparison with other fuzzy multiple criteria analysis methods are as follows: (1) A relative importance of each alternative as well as a utility degree of each alternative is introduced under an interval-valued fuzzy environment to obtain the best alternative' rank among potential alternatives and to indicate, as a percentage, the extent to which one alternative is better or worse than other alternatives regarded for the comparison; (2) the proposed method enables the experts or DMs to provide a reduced criterion determining the overall efficiency of each alternative under an interval-

Table 6 Values under objective criteria

Robots	Load capacity (C_4)	Positioning accuracy±*in* (C_5)	Purchase cost ($\$\times1{,}000$) ($C_6$)
R_1	[(48.5,49);50; (51,52)]	[(0.11,0.12);0.13; (0.14,0.15)]	[(72.5,73);73.5; (74,74.5)]
R_2	[(44,44.5);45; (45.5,46.5)]	[(0.15,0.16);0.17; (0.18,0.19)]	[(69,69.5);70; (71,72)]
R_3	[(43.5,44);45; (46,47.5)]	[(0.16,0.17);0.18; (0.19,0.20)]	[(67.5,68);68.5; (69,70)]

Table 7 Normalized decision matrix

Robots	C_1	C_2	C_3	C_4	C_5	C_6
R_1	[(0.506,0.64);0.773; (0.866,0.933)]	[(0.36,0.506);0.64; (0.746,0.826)]	[(0.325,0.45);0.6; (0.725,0.812)]	[(0.93,0.94);0.96; (0.95,1)]	[(0.55,0.6);0.65; (0.7,0.75)]	[(0.9,0.912);0.918; (0.92,0.93)]
R_2	[(0.346,0.48);0.64; (0.773,0.866)]	[(0.66,0.8);0.906; (0.96,1)]	[(0.55,0.7);0.825; (0.887,0.937)]	[(0.84,0.85);0.86; (0.87,0.89)]	[(0.75,0.8);0.85; (0.9,0.95)]	[(0.93,0.95);0.96; (0.971,0.978)]
R_3	[(0.586,0.746);0.88; (0.946,1)]	[(0.586,0.746);0.88; (0.946,1)]	[(0.625,0.8);0.925; (0.962,1)]	[(0.83,0.84);0.86; (0.88,0.91)]	[(0.8,0.85);0.9; (0.95,1)]	[(0.96,0.97);0.98; (0.99,1)]

Table 8 Weighted normalized decision matrix

Robots	C_1	C_2	C_3	C_4	C_5	C_6
R_1	[(0.35,0.54);0.73; (0.84,0.93)]	[(0.22,0.37);0.54; (0.67,0.77)]	[(0.04,0.07);0.18; (0.33,0.44)]	[(0.72,0.73);0.93; (0.96,1)]	[(0.34,0.40);0.6; (0.67,0.75)]	[(0.16,0.17);0.36; (0.59,0.60)]
R_2	[(0.24,0.41);0.61; (0.75,0.86)]	[(0.41,0.56);0.77; (0.86,0.93)]	[(0.06,0.11);0.24; (0.45,0.51)]	[(0.65,0.66);0.83; (0.86,0.89)]	[(0.46,0.53);0.78; (0.86,0.95)]	[(0.17,0.18);0.38; (0.62,0.63)]
R_3	[(0.41,0.61);0.83; (0.92,1)]	[(0.36,0.55);0.74; (0.85,0.93)]	[(0.07;0.11);0.27; (0.50,0.55)]	[(0.64,0.66);0.83; (0.86,0.91)]	[(0.5,0.56);0.83; (0.91,1)]	[(0.18,0.183);0.39; (0.63,0.65)]

valued fuzzy environment. The generalized criterion can be directly proportional to interval-valued fuzzy relative effect of the values and interval-valued fuzzy weights of criteria on the efficiency of each alternative; (3) the proposed method has the ability to determine a complete ranking of alternatives under an interval-valued fuzzy environment by indicating the position of each alternative, and the ability to deal with criteria of both positive and negative influences and those of quantitative and qualitative natures; and (4) ease of use and understanding of the proposed method under uncertainty so that the interested experts or DMs can easily adopt the method through the group decision-making process.

Furthermore, the results illustrate the applicability and suitability of the proposed IVF-COPRAS method for the evaluation and selection problem. This method can be regarded as an effective subjective and objective integrated decision-making aid.

Although this paper has applied the IVF-COPRAS method to the robot selection problem, the proposed method can be utilized for making an appropriate decision in any other fields of engineering and management problems, such as material selection problem, project selection problem, and supplier selection problem, according to the above-mentioned advantages and benefits. Consequently, the interval-valued fuzzy decision matrix data (criteria values) can be changed and computed for other fields. The weighting (relative importance) of the criteria can be adapted and provided with the requirements of the concerned top managers and depending on their situations. Moreover, the number of potential alternatives (candidates) for consideration can be small or large depending on the experts or DMs' needs to be evaluated and ranked. However, combining the proposed IVF-COPRAS method with the well-known weighting method, such as AHP or linear programming technique for multi-dimensional analysis of preference (LINMAP), for the conflicting criteria can lead to improve and enhance the process of decision making in the real-life situations. This topic can be recommended as the future research in the foregoing fields.

Table 9 Values of $\widetilde{Q}_i$ and N_i

Robots	$\widetilde{Q}_i$	$h\left(\widetilde{Q}_i\right)$	N_i	Rank
R_1	[(1.73,2.18);3.39;(5.82,6.25)]	22.78	90.15	3
R_2	[(1.89,2.32);3.62;(6.07,6.43)]	23.97	94.85	2
R_3	[(2.04,2.55);3.90;(6.26,6.60)]	25.27	100	1

Table 10 Preference order ranking of interval-valued fuzzy TOPSIS method presented by Ashtiani et al. [31]

Robots	$[D_{i1}^{+},D_{i2}^{+}]$	$[D_{i2}^{-},D_{i1}^{-}]$	RC_i^{*}	Rank
R_1	[1.2408, 1.3951]	[2.5413, 3.2079]	0.6792	2
R_2	[1.2365, 1.8974]	[2.312, 3.1021]	0.6320	3
R_3	[1.2751, 1.6579]	[2.9151, 3.3414]	0.6821	1

6 Conclusion

To increase product quality as well as productivity, the robot selection has been regarded as a critical issue for many manufacturing companies. To solve the robot selection problem and take uncertainties into account, we can utilize multiple criteria decision-making (MCDM) approach in an interval-valued fuzzy environment, which can be widely applied in a variety of engineering and management fields. This paper proposed a decision-making method (IVF-COPRAS) based on the complex proportional assessment for solving the robot selection problems. In this method, the performance rating values as well as the weights of criteria were linguistic variables expressed as interval-valued triangular fuzzy numbers. Moreover, we rated the performance of alternatives against subjective criteria via linguistic variables expressed as interval-valued triangular fuzzy numbers. Proposed interval-valued fuzzy complex proportional assessment method is effective and easy to understand. This method constantly enhanced and extended the theory and concept of fuzzy compromise programming based on positive and negative-ideal solutions as well as the fuzzy utility degree. It introduced as a new approach under uncertainty for solving the selection and evaluation problems. Although the presented IVF-COPRAS method was applied for the robot selection, it can be applied for making a best decision in any other fields of engineering and management, particularly in other manufacturing decision-making problems.

Acknowledgments The authors gratefully acknowledge comments and suggestions made by the editor-in-chief and anonymous referees to improve the primary version of this paper.

Appendix

An interval-valued fuzzy set $\widetilde{A}$ on $\mathfrak{R}$ is given by $\widetilde{A} \overset{\Delta}{=} \left\{\left(x, \left[\mu_{\tilde{A}^L}(x), \mu_{\tilde{A}^U}(x)\right]\right\}, x \in \mathfrak{R}$, when $\mu_{\tilde{A}^L}(x), \mu_{\tilde{A}^U}(x) \in [0,1]$ and $\mu_{\tilde{A}^L}(x) \leq \mu_{\tilde{A}^U}(x) \forall x \in \mathfrak{R}$ are denoted as $\widetilde{A} = \left[\widetilde{A}^L, \widetilde{A}^U\right]$ [17]. This means that the grade of membership of x belongs to the interval $\left[\mu_{\tilde{A}^L}(x), \mu_{\tilde{A}^U}(x)\right]$, the least grade of membership at x is $\mu_{\tilde{A}^L}(x)$, and the greatest grade of membership at x is $\mu_{\tilde{A}^U}(x)$ [49]. Let

$$\mu_{\widetilde{A}^L}(x) = \begin{cases} \lambda(x-p)\big/(q-p), & p \leq x \leq q, \\ \lambda(r-x)\big/(r-q), & q \leq x \leq r, \\ 0, & otherwise \end{cases}.$$

Then, $\widetilde{A}^L = (p, q, r; \lambda)$. Let

$$\mu_{\widetilde{A}^U}(x) = \begin{cases} \rho(x-e)\big/(q-e), & e \leq x \leq q, \\ \rho(h-x)\big/(h-q), & q \leq x \leq h, \\ 0, & otherwise \end{cases}.$$

Then, $\widetilde{A}^U = (e, q, h; \rho)$. Here, $0 < \lambda \leq \rho \leq 1, e < p < q < r < h$. Thus, we have i-v fuzzy set $\widetilde{A} \overset{\Delta}{=} \left\{\left(x, \left[\mu_{\tilde{A}^L}(x), \mu_{\tilde{A}^U}(x)\right]\right\}, \quad x \in \mathfrak{R}$. We denote $\widetilde{A} = [(p, q, r; \lambda), (e, q, h; \rho)] = \left[\widetilde{A}^L, \widetilde{A}^U\right]$. $\widetilde{A}$ is called a level (λ, ρ) i-v fuzzy numbers as shown in Fig. 1. Let

$$F_{IV}(\lambda, \rho) = \left\{[(p, q, r; \lambda), (e, q, h; \rho)] \middle| \forall e < p < q < r < h\right\}, \quad 0 < \lambda \leq \rho \leq 1$$

or

$$F_{IV}(\lambda, \rho) = \left\{[(p, r; \lambda); q; (e, h; \rho)] \middle| \forall e < p < q < r < h\right\}, \quad 0 < \lambda \leq \rho \leq 1.$$

Given two interval-valued fuzzy numbers $\widetilde{A} = \left[\widetilde{A}^L, \widetilde{A}^U\right]$ and $\widetilde{B} = \left[\widetilde{B}^L, \widetilde{B}^U\right]$, according to [49, 50], we have:

Definition A.1 If $\circ \in (+, -, \times, \div)$, then $\widetilde{A} \circ \widetilde{B} = \left[\widetilde{A}^L \circ \widetilde{B}^L; \widetilde{A}^U \circ \widetilde{B}^U\right]$, for a positive non-fuzzy number (v), $v \circ \widetilde{A} = \left[v \circ \widetilde{A}^L; v \circ \widetilde{A}^U\right]$.

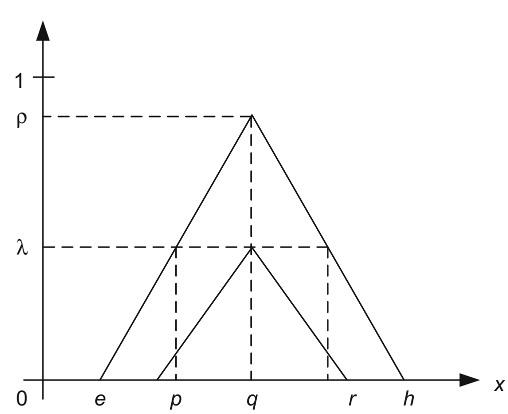

Fig. 1 Level (λ, ρ) i-v fuzzy

Definition A.2 Let $\widetilde{A}$ and $\widetilde{B}$ be two interval-valued fuzzy numbers. $\widetilde{A}$ and $\widetilde{B}$ can then be represented as follows:

$$\tilde{A} = [(a_1, a_2)a_3; (a_4, a_5)] \text{ and}$$

$$\tilde{B} = [(b_1, b_2); b_3; (b_4, b_5)] Let$$

$$h\left(\tilde{A}\right) = \frac{a_1 + a_2 + 2a_3 + a_4 + a_5}{6},$$

$$h\left(\tilde{B}\right) = \frac{b_1 + b_2 + 2b_3 + b_4 + b_5}{6},$$

We say $\tilde{A} > \tilde{B}$ *if* $h\left(\tilde{A}\right) > h\left(\tilde{B}\right)$.

References

1. Rao RV (2007) Decision making in the manufacturing environment using graph theory and fuzzy multiple attribute decision making methods. Springer, London
2. Knott K, Getto RD (1982) A model for evaluating alternative robot systems under uncertainty. Int J Prod Res 20:155–165
3. Agrawal VP, Kohli V, Gupta S (1991) Computer aided robot selection: the multiple attribute decision making approach. Int J Prod Res 29:1629–1644
4. Liang GH, Wang M-JJ (1993) A fuzzy multi-criteria decision making approach for robot selection. Robot Comput Integr Manuf 10:267–274
5. Goh CH (1997) Analytic hierarchy process for robot selection. J Manuf Syst 16:381–386
6. Parkan C, Wu ML (1999) Decision-making and performance measurement models with applications to robot selection. Comput Ind Eng 36:503–523
7. Khouja MJ, Kumar RL (1999) An options view of robot performance parameters in a dynamic environment. Int J Prod Res 37:1243–1257
8. Bragilia M, Petroni A (1999) Evaluating and selecting investments in industrial robots. Int J Prod Res 37:4157–4178
9. Layek AM, Resare LJ (2000) Algorithm based decision support system for the concerted selection of equipment in machining/assembly cells. Int J Prod Res 38:323–339
10. Chu TC, Lin YC (2003) A fuzzy TOPSIS method for robot selection. Int J Adv Manuf Technol 21:284–290
11. Bhangale PP, Agrawal VP, Saha SK (2004) Attribute based specification, comparison and selection of a robot. Mech Mach Theory 39:1345–1366
12. Karsak EE, Ahiska SS (2005) Practical common weight multi-criteria decision-making approach with an improved discriminating power for technology selection. Int J Prod Res 43(8):1537–1554
13. Rao RV, Padmanabhan KK (2006) Selection, identification and comparison of industrial robots using digraph and matrix methods. Robot Comput Integr Manuf 22:373–383
14. Chatterjee P, Manikrao Athawale V, Chakraborty S (2010) Selection of industrial robots using compromise ranking and outranking methods. Robot Comput Integr Manuf 26:483–489
15. Sambuc R (1975) Fonctions f-floues. Application a'l'aide au diagnostic en pathologie thyroidienne. PhD Thesis, University of Marseille
16. Grattan-Guinness I (1975) Fuzzy membership mapped onto interval and many-valued quantities. Z Math Logik Grundlag Mathe 22:149–160
17. Gorzalczany MB (1987) A method of inference in approximate reasoning based on interval-valued fuzzy sets. Fuzzy Sets Syst 21:1–17
18. Turksen IB (1986) Interval-valued fuzzy sets based on normal forms. Fuzzy Sets Syst 20:191–210
19. Cornelis C, Deschrijver G, Kerre EE (2006) Advances and challenges in interval-valued fuzzy logic. Fuzzy Sets Syst 157:622–627
20. Karnik NN, Mendel JM (2001) Operations on type-2 fuzzy sets. Fuzzy Sets Syst 122:327–348
21. Wang G, Li X (1998) The applications of interval-valued fuzzy numbers and interval-distribution numbers. Fuzzy Sets Syst 98:331–335
22. Bustince H (1994) Conjuntos Intuicionistas e Intervalo valorados Difusos: Propiedades y Construccion, Relaciones Intuicionistas Fuzzy. Thesis, Universidad Publicade Navarra
23. Turksen IB (1992) Interval-valued fuzzy sets and compensatory. Fuzzy Sets Syst 51:295–307
24. Turksen IB (1996) Interval-valued strict preference with Zadeh triples. Fuzzy Sets Syst 78:183–195
25. Mustajoki J, Hamalainen RP, Salo A (2005) Decision support by interval SMART/SWING—incorporating imprecision in the SMART and SWING methods. Decis Sci 36(2):317–339
26. Halouani N, Chabchoub H, Martel JM (2009) PROMETHEE-MD-2T method for project selection. Eur J Oper Res 195(3):841–849
27. Vahdani B, Hadipour H (2011) Extension of ELECTRE method based on interval-valued fuzzy sets. Soft Comput 15(3):569–579
28. Vahdani B, Hadipour H, Sadaghiani JH, Amiri M (2010) Extension of VIKOR method based on interval-valued fuzzy sets. Int J Adv Manuf Technol 47(9–12):1231–1239
29. Vahdani B, Tavakkoli-Moghaddam R, Mousavi SM, Ghodratnama A (2013) Soft computing based on new fuzzy modified multi-criteria decision making method. Appl Soft Comput 13:165–172
30. Yao JS, Yu MM (2004) Decision making based on statistical data, signed distance and compositional rule of inference. Int J Uncertain Fuzzy Knowl-Based Syst 12(2):161–190
31. Ashtiani B, Haghighirad F, Makui A, Montazer G (2009) Extension of fuzzy TOPSIS method based on interval-valued fuzzy sets. Appl Soft Comput 9:457–461
32. Mousavi SM, Vahdani B, Tavakkoli-Moghaddam R, Ebrahimnejad S, Amiri M (2013) A multi-stage decision making process for multiple attributes analysis under an interval-valued fuzzy environment. Int J Adv Manuf Technol 64:1263–1273
33. Ebrahimnejad S, Mousavi SM, Seyrafianpour H (2010) Risk identification and assessment for build-operate-transfer projects: a fuzzy multi attribute decision making model. Expert Syst Appl 37:575–586
34. Mousavi SM, Torabi SA, Tavakkoli-Moghaddam R (2013) A hierarchical group decision-making approach for new product selection in a fuzzy environment. Arab J Sci Eng 38(11):3233–3248
35. Ebrahimnejad S, Mousavi SM, Tavakkoli-Moghaddam R, Heydar M (2012) Evaluating high risks in large-scale projects using an extended VIKOR method under a fuzzy environment. Int J Ind Eng Comput 3:463–476
36. Mousavi SM, Jolai F, Tavakkoli-Moghaddam R (2013) A fuzzy stochastic multi-attribute group decision-making approach for selection problems. Group Decis Negot 22(2):207–233
37. Opricovic S, Tzeng GH (2004) Compromise solution by MCDM methods: a comparative analysis of VIKOR and TOPSIS. Eur J Oper Res 156:445–455
38. Opricovic S, Tzeng GH (2007) Extended VIKOR method in comparison with outranking methods. Eur J Oper Res 178(2):514–529
39. Chatterjee P, Manikrao Athawale V, Chakraborty S (2011) Materials selection using complex proportional assessment and evaluation of mixed data methods. Mater Des 32:851–860

40. Zavadskas EK, Turskis Z, Tamosaitiene J, Marina V (2008) Multicriteria selection of project managers by applying grey criteria. Technol Econ Dev Econ 14(4):462–477
41. Triantaphyllou E (2000) Multi-criteria decision making methods: a comparative study. Kluwer Academic, London
42. Tavakkoli-Moghaddam R, Mousavi SM, Heydar M (2011) An integrated AHP-VIKOR methodology for plant location selection. Int J Eng Trans B Appl 24(2):127–137
43. Vahdani B, Mousavi SM, Tavakkoli-Moghaddam R (2011) Group decision making based on novel fuzzy modified TOPSIS method. Appl Math Model 35:4257–4269
44. Mousavi SM, Tavakkoli-Moghaddam R, Heydar M, Ebrahimnejad S (2013) Multi-criteria decision-making for plant location selection: an integrated Delphi-AHP-PROMETHEE methodology. Arab J Sci Eng 38:1255–1268
45. Mousavi SM, Makui A, Raissi S, Mojtahedi SMH (2012) A multi-criteria decision-making approach with interval numbers for evaluating project risk responses International. J Eng Trans B Appl 25(2):117–124
46. Zavadskas E, Kaklauskas A (1996) Multiple criteria analysis of buildings (in Lithuanian). Technika, Vilnius
47. Zavadskas EK (1987) Complex estimation and choice of resource saving decisions in construction. Mokslas, Vilnius
48. Zadeh LA (1975) The concept of a linguistic variable and its application to approximate reasoning. Inf Sci 8:301–357, (a) 199–249
49. Chiang J (2001) Fuzzy linear programming based on statistical confidence interval and interval-valued fuzzy set. Eur J Oper Res 129:65–86
50. Grzegorzewski P (2004) Distances between intuitionistic fuzzy sets and/or interval valued fuzzy sets based on the Hausdorff metric. Fuzzy Sets Syst 148:319–328