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Research Article

An extension of fuzzy P-control chart based on α-level fuzzy midrange

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Abstract
Control charts are one of the most important tools in statistical process control that lead to improve quality processes and ensure required quality levels. In traditional control charts, all data should be exactly known, whereas there are many quality characteristics that cannot be expressed in numerical scale, such as characteristics for appearance, softness, and color. Fuzzy sets theory is powerful mathematical approach to analyze uncertainty, ambiguous and incomplete that can linguistically define data in these situations. Fuzzy control charts have been extended by converting the fuzzy sets associated with linguistic or uncertain values into scalars regarded as representative values. In this paper, we develop a new fuzzy control chart for monitoring attribute quality characteristics based on α-level fuzzy midrange approach. Finally, the performance and comparative results of the proposed fuzzy control chart is measured in terms of average run length (ARL) by Mont Carlo simulation.

Keywords: Attribute control chart, Fuzzy sets theory, α-level fuzzy midrange, Average run length.

1 Introduction

Nowadays competitive global market enhancing quality is an important business strategy. In this regard, improving tools of monitoring the quality process has become inevitable. Statistical process control (SPC) is a major tool in many manufacturing environment for implementing quality improvement programs. The SPC process includes observation, evaluation, diagnosis, decision and implementation. Control charts are widely applied in SPC tools. Despite the first control charts proposed during 1920s by Shewhart, they still have an extensive application especially in manufacturing processes in industrial applications.
Control charts were designed to monitor a process and detect shifts in mean and variance of quality characteristics to assure that the processes are performing in an acceptable manner. Two main types of control charts include variable and attribute control charts. The first is used to monitor measurable characteristics on numerical scales. Quality characteristics cannot be easily represented in numerical form monitored by second. In contrast to variable control charts, attribute control charts could monitor more

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than one quality characteristic simultaneously and need less cost and time for inspections. However the observation of these control charts accompany with ambiguous and vague. In classical *P*-control charts, products are clearly categorized as conformed and non-conformed. In many situations, binary classification may not be appropriate since they have several intermediate levels and the necessity to apply mathematical powerful tool in order to increase the performance of control charts. Hence, recently fuzzy control charts have been extended to analyze uncertainty, ambiguous and incomplete or linguistically defined data. Fuzzy sets convert associated linguistic or uncertain values into scalars regarded as representative values.

The fuzzy sets theory was first proposed by Zadeh [1]. In the literature, different attempts were done to construct control charts according to fuzzy logic and fuzzy sets theory. Kanagawa et al. [2] designed control charts to monitor mean and deviation based on linguistic data. In this scope, a comprehensive literature review for fuzzy quality control made by Guiffrida and Nagi [3]. Wang and Raz [4] illustrate two approaches for constructing variable control charts based on linguistic data. Afterwards, Raz and Wang [5] assigned fuzzy sets to each linguistic term in order to create and design control charts for linguistic data. Gülbay and Kahraman [6] constructed α-level fuzzy control charts for attributes data to represent the ambiguous of the data and strange of the inspection. Also, they [7] presented an approach that does not require the use of any fuzzy transformation methods to compare fuzzy linguistics data, namely direct fuzzy approach (DFA). Engin et al. [8] developed a fuzzy model for attribute control charts in multi-stage processes. Taleb and Limam [9] compared different procedures for constructing attribute control charts with the help of fuzzy and probability theory. In addition, they [10] presented fuzzy multi-nominal control charts to monitor multi-variate attribute processes with linguistic data. Shu and Wu [11] applied resolution identity to construct the control limits of fuzzy *P*-control chart using fuzzy data. Sorooshian [12] proposed monitoring attribute quality characteristic with consideration of uncertainty and ambiguous. An attribute control chart was proposed for monitoring the mean of variables by Wu and Jiao [13]. Also, Faraz et al. [14] constructed a fuzzy statistical control chart that explained existing fuzziness in data by considering variability between observations. The structure of the α-level fuzzy midrange for control chart have been proposed for triangular and trapezoidal membership functions by Sevil [15]. Moameni et al. [16] appraised the effect of measurement error on the effectual of the fuzzy control chart. Recently, Wan [17] focused on multi-attribute group decision making problems in which the attribute values are expressed with trapezoidal intuitionistic fuzzy numbers, in which they are solved by developing a new decision method.

In real-applications, there are many cases with uncertainty, ambiguous and incomplete or linguistically defined data. Obviously, mentioned data effect on the performance of attribute control chart. Hence, it is necessary to use a new approach that increases flexibility in range of observation whereas improve the performance of attribute control chart in detection of assignable cause. We consider a new method for control charts to transform fuzzy sets into scalars based on α-level fuzzy midrange.

The reminder of this paper organized as follows: in the next section, we discuss about transformation techniques. In section 3, firstly we review crisp *P*-control charts; then, α-level fuzzy are developed for observations in these control charts. Finally, α-level fuzzy midrange is presented for *P*-control charts. In section 4, the performance and advantage of the proposed fuzzy control chart is measured in terms of average run length (ARL) with respect to traditional control chart via Mont Carlo simulation. Finally, conclusions and future research are presented in section 5.

2 Fuzzy transformation methods

To construct standard format of control charts and facilitate the plotting of observations on the chart, we need to convert the fuzzy sets associated with the linguistic or uncertain values into scalars regarded representative values [4]. Notably that in a fuzzy environment each sample or subgroup can be commonly represented by triangular fuzzy numbers (*a, b, c*). There are several methods in this area, four commonly used methods are known as: fuzzy median, fuzzy average, fuzzy mode and α -level fuzzy midrange as below:

- Fuzzy median: the point which partitions the curve under membership function of a fuzzy set into equal regions satisfying the following equations where *a* and *b* are the end points in the base variable of fuzzy set *F* such that *a* < *b*:

$$\int_{a}^{f_{med}} \mu_F(x)dx = \int_{f_{med}}^{b} \mu_F(x)dx = \frac{1}{2}\int_{a}^{b} \mu_F(x)dx. \tag{2.1}$$

- Fuzzy average: based on Zadeh [1], the fuzzy average is:

$$f_{avg} = Av(x:F) = \frac{\int_0^1 x\mu_F(x)dx}{\int_0^1 \mu_F(x)dx} \tag{2.2}$$

- Fuzzy mode: The fuzzy mode of a fuzzy set *F* is the value of the base variable where the membership functions equal to 1. This is stated as below:

$$f_{\mathrm{mod}} = \{\mathrm{x} : \mu_F(x) = 1\} \tag{2.3}$$

- α-level fuzzy midrange: it shows the midpoint of the ends for the α-level cut. An α-level cut given by A_α is a non-fuzzy set which comprise all elements whose membership is greater than or equal to α. The α-level fuzzy midrange is determined for a triangular fuzzy set as the midpoint of the crisp interval that divides the set into two subsets. One subset is related to all the values that have a membership larger than or equal to α in the original set. The other subset concludes all the membership less than α. The midpoint of the ends of the α-level cuts is a definition for α-level fuzzy midrange $f_{mr}(\alpha)$. An α-level cut, given by A_α, is a non-fuzzy set if a_α and c_α are the end points of A_α [4]. Thus, α-level fuzzy midrange is calculated as follows:

$$f_{mr}(\alpha) = \frac{1}{2}(a_\alpha + c_\alpha), \tag{2.4}$$

In Figure 1, α-cut on a sample by triangular fuzzy numbers is represented; also, α-level fuzzy midrange of sample *j*, ($S^{\alpha}_{mr,j}$) is determined by Eq. (2.5) [1]:

$$S^{\alpha}_{mr,j} = \frac{(a_j + c_j) + \alpha[(b_j - a_j) - (c_j - b_j)]}{2} \tag{2.5}$$

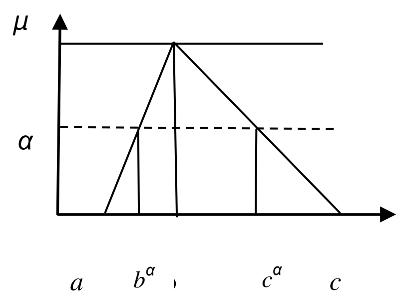

Figure 1: Representation of α-cut on a sample by triangular fuzzy numbers [4]

3 Fuzzy *P*-control chart based on range

In statistical quality control, we apply *P*-control chart to monitor fraction rejected units of products. It shows the number of nonconforming items, exist in entire process. In the traditional approach the formulation of upper bound and lower bound of *P*-control charts were based on crisp data and calculated by given following equations:

$$CL = \bar{P},$$
$$LCL = \bar{P} - 3\sqrt{\frac{\bar{P}(1-\bar{P})}{n}},$$
$$UCL = \bar{P} + 3\sqrt{\frac{\bar{P}(1-\bar{P})}{n}}. \tag{3.6}$$

The fraction of nonconforming product for each sample plots on the control limits. If some samples plot outside the control limits, it shows that the process should be stopped to determine assignable causes. In this study, we consider number of defects as triangular fuzzy numbers that represented by (D_a,D_b, D_c) for each fuzzy sample. *CL* and *UCL, LCL* represented the center line and control limits of fuzzy *P*-control charts respectively and they are triangular fuzzy sets; then, they are determined by the following equations:

$$C\tilde{L} = (CL_a, CL_b, CL_c) = (\bar{P}_a, \bar{P}_b, \bar{P}_c), \tag{3.7}$$

$$(\bar{P}_a, \bar{P}_b, \bar{P}_c) = (\frac{\sum D_a}{mn}, \frac{\sum D_b}{mn}, \frac{\sum D_c}{mn}), \tag{3.8}$$

$$LCL = (LCL_a, LCL_b, LCL_c) = (\bar{P}_a - 3\sqrt{\frac{\bar{P}_a(1-\bar{P}_a)}{n}}, \bar{P}_b - 3\sqrt{\frac{\bar{P}_b(1-\bar{P}_b)}{n}}, \bar{P}_c - 3\sqrt{\frac{\bar{P}_c(1-\bar{P}_c)}{n}}), \tag{3.9}$$

$$UCL = (UCL_a, UCL_b, UCL_c) = (\bar{P}_a + 3\sqrt{\frac{\bar{P}_a(1-\bar{P}_a)}{n}}, \bar{P}_b + 3\sqrt{\frac{\bar{P}_b(1-\bar{P}_b)}{n}}, \bar{P}_c + 3\sqrt{\frac{\bar{P}_c(1-\bar{P}_c)}{n}}), \tag{3.10}$$

where n and m display the fuzzy sample size and the number of subgroups, respectively.

3.1. *α*-Cut fuzzy *P*-control chart

In this paper, we construct fuzzy *P*-control charts with the help of α-cut method. The interpretation of these charts is the same as mentioned in the previous section. By applying α-cuts on fuzzy sets, the values of center line are determined as follows:

$$C\tilde{L} = (\mathrm{CL}_a^{\alpha}, \mathrm{CL}_b, \mathrm{CL}_c^{\alpha}) = (\bar{P}_a^{\alpha}, \bar{P}_b, \bar{P}_c^{\alpha}), \tag{3.11}$$

Similarly, α-cut fuzzy *P*-control chart limits based on ranges can be stated as follows:

$$(\bar{P}_a^{\alpha}, \bar{P}_b, \bar{P}_c^{\alpha}) = (\frac{\sum D_a^{\alpha}}{mn}, \frac{\sum D_b}{mn}, \frac{\sum D_c^{\alpha}}{mn}), \tag{3.12}$$

where in $\bar{P}_a^{\alpha}$ and $\bar{P}_c^{\alpha}$ as follows:

$$\bar{P}_a^{\alpha} = \bar{P}_a + \alpha(\bar{P}_b - \bar{P}_a), \tag{3.13}$$

$$\bar{P}_c^{\alpha} = \bar{P}_c - \alpha(\bar{P}_c - \bar{P}_b). \tag{3.14}$$

3.2. α -Level fuzzy midrange for *P*-control chart

As aforementioned α-level fuzzy midrange is one of four transformation techniques that have used to design the fuzzy control limits. These control limits are employed to give a decision whether the process is in the control or out of the control. In this study, an α-level fuzzy midrange is used as the fuzzy transformation method while calculating the central line and control limits are as follows:

$$CL_{mr-p}^{\alpha} = \frac{1}{2}(\bar{P}_a^{\alpha} + \bar{P}_{\text{c}}^{\alpha}), \tag{3.15}$$

$$LCL_{mr-p}^{\alpha} = CL_{mr-p}^{\alpha} - 3\sqrt{\frac{CL_{mr-p}^{\alpha}(1 - CL_{mr-p}^{\alpha})}{n}}, \tag{3.16}$$

$$UCL_{mr-p}^{\alpha} = CL_{mr-p}^{\alpha} + 3\sqrt{\frac{CL_{mr-p}^{\alpha}(1 - CL_{mr-p}^{\alpha})}{n}}. \tag{3.17}$$

As mentioned in section 2, a definition of α-level fuzzy midrange of sample j for fuzzy P-control chart is:

$$S_{mr-p,j}^{\alpha} = \frac{(D_{a_j} + D_{c_j}) + \alpha[(D_{b_j} - D_{a_j}) - (D_{c_j} - D_{b_j})]}{2n}. \tag{3.18}$$

Then, the condition of process control for each sample can be defined as:

$$\text{Process control} = \begin{Bmatrix} incontrol & \text{for LCL}_{mr-p}^{\alpha} \le S_{mr-p,j}^{\alpha} \le U\text{CL}_{mr-p}^{\alpha} \\ out-control & for\ otherwise \end{Bmatrix}$$

4 Results and discussion

The aim of this section is to comparing the proposed fuzzy P-control chart with traditional shewhart control chart in order to show that the proposed approach has better performance for detection shifts. ARL is the average of the number of samples which should occur before a sample shows the out-of-control condition. If the process's observations are not auto-correlated, ARL could be calculated for every type of control charts to appraise their ability [19]. Hence, in this paper, the ARL is applied as an evaluation criterion to compare the performance of proposed fuzzy P-control chart with the classical approach. It is pointed out that there are two different ARLs as follows:

1) In the control state, ARL is shown by ARL_0 and it is the number of samples' average which should occur before a sample shows an out-of-control condition when the process is in fact in the state of in-control.

2) An ARL is shown by $ARL_1$when the process is out-of-control. In reality, ARL_1 is the number of samples' average, taking place until a point shows an out-of-control condition when the process is in fact out-of-control. ARL_1 could be calculated by $ARL_1 = \frac{1}{1-\beta}$ where β is the type II error and shows the probability of not detecting a shift with the first point after the occurrence of a shift in the process [12].

Usually, for traditional control charts with three sigma control limits, the probability of type I error considered 0.0027 which is the probability of being out-of-control a point when the process is in control. When the process is in control the ARL means that averagely after each 370 points, a point shows an alarm of out-of-control when the process is in fact in control state.
When the process is in out-of-control state, ARL is shown by ARL_1. We consider a numerical example to represent the performance of the presented method. To generate the fuzzy and crisp data and running the simulation, MATLAB release R2009a has been used. Firstly, we fix control limit to obtain error type I equal to 0.005 in phase I. Afterwards, we shift the parameter process and calculate ARL_1 under predefined process shifts. A simulation is performed for their sensitivity to the process shift with $\alpha = 0.99$. The results of the proposed approach for *P*-control chart are shown in Table 1, and they compare with traditional control chart. By considering Table 1, it can be concluded that the presented fuzzy approach has smaller ARL_1 than traditional control chart under different shift. Hence, the performance of the fuzzy *P*-control chart has been significantly improved to detect assignable causes. On the other hand, the flexibility of the decision makers increases by consideration the fuzzy data. Also, as shown in Table 1, the magnitude shifts increases and the power of control chart is then augmented.

Table 1: Comparison between ARL_1 in the proposed fuzzy *P*-chart and traditional *P*-chart.

Shift	**ARL**	
	Proposed fuzzy chart	**Traditional chart**
0.041	**86.5362**	**86.5593**
0.083	**31.6677**	**31. 1604**
0.125	**12.9439**	**12.9951**
0.1666	**6.4157**	**6.4599**
0.208	**3.6407**	**3.6889**
0.214	**2.3537**	**2.3591**
0.25	**1.6947**	**1.7051**
0.29	**1.3398**	**1.3469**
0.33	**1.0564**	**1.1135**

5 Conclusion and future research

Control charts have extensive applications to find shifts in the process and indicate abnormal process condition. One of the most important SPC tools is attribute control chart that monitors quality characteristics. Some causes such as mental inspection, incomplete data and human judgments in quality characteristic that lead to exist some level of vagueness and uncertainty in attribute control chart, in these situations it is better to apply fuzzy set theory for control charts. Thus, in this paper, we developed a fuzzy *P*-chart based on α-level fuzzy midrange to monitor attribute quality characteristic. Results of the comparison study by using ARL criterion showed that the proposed approach had high performance and could detect shifts in the process faster than traditional control chart. Considering trapezoidal fuzzy numbers and applying other control charts such as exponentially weighted moving average can be interesting topics for future research.

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