16th Iranian International Industrial Engineering Conference

22-23 January2020

Alzahra University of Tehran

The performance of variable sampling interval EWMA control chart with measurement errors

Zeynab Hassani[1], Amirhossein Amiri[2*]

[1]Department of Industrial Engineering, Shahed University, Tehran, Iran; zeynab.hasani@shahed.ac.ir

[2]Department of Industrial Engineering, Shahed University, Tehran, Iran; amiri@shahed.ac.ir

* Corresponding author: Amirhossein Amiri

Abstract

The effect of measurement errors on the performance of the most adaptive Shewhart control charts is investigated. In this paper, the effect of measurement errors on the performance of the Variable Sampling Interval (VSI) EWMA control chart is investigated. For this aim, the covariate errors model and Markov chain method are used to compute the evaluation indices. Furthermore, the effect of taking multiple measurements on the performance of the VSI EWMA is evaluated. Also, the performance of the VSI EWMA and FSI EWMA control charts in presence of measurement errors is compared in terms of Average Time to Signal (*ATS*). At the end, an example is provided to show the application of the proposed VSI EWMA control chart in reducing the effect of measurement errors.

Keywords: Adaptive control chart; Exponentially Weighted Moving Average (EWMA); Markov chain method Measurement errors; Variable sampling interval (VSI).

1. Introduction

Shewhart control charts are effective in detecting large mean shifts of quality characteristics. But, these control charts have poor performance in detecting small shifts. While, the EWMA control chart outperforms Shewhart control charts in detecting the small and moderate shifts. To improve the performance of control charts in detecting shifts, the adaptive control charts are proposed. in which, at least one of parameters of control chart (sample size, sampling interval, and the control limit coefficient) is variable throughout the process.

The Variable Sample Size (VSS) $\bar{X}$ control chart has been proposed by Prabhue et al. [1] and Costa [2]. Reynols [3] introduced charts with the Variable Sampling Interval (VSI). The VSSI $\bar{X}$ control chart which combines the VSI and VSS has been investigated by Prabhue et al. [4] and Costa [5]. Also, Costa [6] proposed a Variable Parameter (VP) $\bar{X}$ control chart. Moreover, a review paper about adaptive control charts is provided by Tagaras [7]. Zimmer et al. [8] provided guidelines for application of the adaptive control charts.

All of above researches have shown the performance of adaptive Shewhart control charts in detecting mean shifts in quality characteristics. Also, the EWMA control chart has better performance than the Shewhart charts in detecting small mean shifts. Hence, Saccucci et al. [9] presented the VSI EWMA control chart and compared it with Fixed Sampling Interval (FSI) EWMA control chart scheme. Reynolds and Arnold [10] investigated the VSI EWMA and VSS EWMA control charts. The VSI EWMA control chart combined with the R and S^2 control chart has been presented by Castagliola et al. [11,12], respectively. The VSI EWMA control chart based the *t* distribution is investigated by Kazemzadeh et al. [13]. Tran et al. [14] presented the VSI EWMA median control chart. Amiri et al. [15] and Ugaz et al. [16] designed an Adaptive EWMA (AEWMA) control chart. Also, Capizzi and

Masaratto [17] investigated the efficiency of the AEWMA median control chart scheme. Ong et al. [18] presented the performance of VSI EWMA with parameter estimation.

All of these studies have not considered the measurement errors in the processes, while in the real world applications, the measurement errors are available, which have negative effect on the performance of control charts. The measurement errors has been introduced by Benet [19] based on the model $X = Y + \varepsilon$, where X is the observed value, Y is the true value of the quality characteristic and ε is the random measurement errors. Also, Linna and Woodall [20] applied $X = A + BY + \varepsilon$, which is a covariate error model between Y and X and A and B are constant and known parameters. After introducing the covariate error model by Linna and Woodall [20], many researchers investigated the effect of measurement errors on the performance of the various control chart such as Linna et al.[21], Kanazuka [22], Costa and Costagliola [23], Hu et al. [24,25,26], Sabahno and Amiri [27], Sabahno et al. [28,29,30], Maleki et al. [31], Amiri et al.[32], Ghashghaei et al.[33], Maravelakis et al.[34], Stemnan and Weish[35], Haq et al.[36], Yang et al. [37], Abbasi [38,39], Tang et al.[40] and Salmasnia et al.[41]. Maleki et al. [42] provided a survey concerning recent contributions in the field of measurement errors in statistical process monitoring. Considering this survey, the effect of measurement errors on the VSI EWMA control chart has never been investigated. Hence, in this study, we use the linear covariate error model, which introduced by Linna and Woodall [20]. Then, we investigate the effect of measurement errors on the performance the VSI EWMA control chart. Also, we consider the effect of the constant B as well as the effect of the number of multiple measurements on the performance of the VSI EWMA control chart.

The structure of this paper is as follows: the linearly covariate error model is described in the next section. The VSI EWMA control chart with measurement errors is introduced in section 3. A modified Markov chain for computing the performance measures of the VSI EWMA control chart is presented in section 4. In the following, the effect of measurement errors on the performance of the VSI EWMA control chart is investigated in section 5. A comparison between the VSI EWMA and FSI EWMA chart with measurement errors is given in section 6. An illustrative example is presented in section 7. Finally, conclusion and suggestions for future research are given in section 8.

2. Linearly covariate error model

We assume that Y_{ij} is independent observation of the quality characteristic where $i = 1, 2, \ldots$ and $j = 1, 2, \ldots, n$. Y_{ij} follows the $N\left(\mu_0 + \delta\sigma_0, \sigma_0^2\right)$ distribution, where μ_0 and σ_0 are in-control mean and standard deviation, respectively and both are assumed known and δ is the magnitude of the standardized mean shift ($\delta = 0$ in the case of in-control). Also the Y_{ij} can not be directly observed, but can estimated by using X_{ijk} , where $K = 1, 2, \ldots, m$ is the index of the number of measurements. X_{ijk} is related to Y_{ij} by using a linearly covariate model as follows:

$$X_{ijk} = A + BY_{ij} + \varepsilon_{ijk}, \quad (1)$$

where A and B are constants and known, and ε_{ijk} is the random error term which follows $N\left(0, \sigma_M^2\right)$ distribution. The sample mean $\bar{X}_i$, for each time i is:

$$\bar{X}_i = \frac{1}{mn} \sum_{j=1}^{n} \sum_{k=1}^{m} X_{ijk} \,. \quad (2)$$

From (1) and (2), we have

$$\bar{X}_i = A + \frac{1}{n}\left(B\sum_{j=1}^{n} Y_{ij} + \frac{1}{m}\sum_{j=1}^{n}\sum_{k=1}^{m}\varepsilon_{ijk} \right). \tag{3}$$

The mean and the variance of $\bar{X}_i$ can be obtained as:

$$\mathrm{E}\left(\bar{X}_i\right) = A + B\left(\mu_0 + \delta\sigma_0\right), \tag{4}$$

$$Var\left(\bar{X}_i\right) = \frac{1}{n}\left(B^2\sigma_0^2 + \frac{\sigma_M^2}{m} \right). \tag{5}$$

3. VSI EWMA control chart with measurement errors

When the sampling interval is allowed to vary throughout the sampling process, the control chart for this scheme is called a VSI control chart. In this paper, we assume that there are two types of sampling interval ($t_1 < t_2$). Since Y_{ij} follows the normal distribution, based on (1) X_{ijk} has a normal distribution. We standardize $\bar{X}_i$ as follows:

$$U_i = \frac{\bar{X}_i - \mu_{0_{\bar{X}_i}}}{\sigma_{\bar{X}_i}}, \tag{6}$$

where $\sigma_{\bar{X}_i}$ is the standard deviation and $\mu_{0_{\bar{X}_i}}$ is the in-control expected value of $\bar{X}_i$. Hence, $U_i \sim N(0,1)$ and it is used in EWMA statistic as follows:

$$Z_i = \lambda U_i + (1-\lambda) Z_{i-1}, \tag{7}$$

where $Z_i \sim N\left(0, \frac{\lambda}{2-\lambda}\right)$, Hence, the steady state upper and lower control limits of the VSI EWMA control chart are obtained by using (8) and (9), respectively:

$$UCL = L\sqrt{\frac{\lambda}{2-\lambda}}, \tag{8}$$

$$LCL = -L\sqrt{\frac{\lambda}{2-\lambda}}. \tag{9}$$

Similarly, the steady state upper and lower warning limits, respectively, are:

$$UWL = W\sqrt{\frac{\lambda}{2-\lambda}}, \tag{10}$$

$$LWL = -W\sqrt{\frac{\lambda}{2-\lambda}}, \tag{11}$$

where $0 < W < L$. The VSI strategy for choosing the next sampling interval is as follows::

- If $Z_i \in \Omega_1 = [LWL, UWL]$, the process is identified as "in-control", and the long sampling interval t_2 is taken for the next sampling period.

- If $Z_i \in \Omega_2 = [LCL, LWL) \cup (UWL, UCL]$, the process is also identified as "in-control", but the small sampling interval t_1 is chosen for the next sampling period.

- If $Z_i \notin \Omega_3 = [LCL, UCL]$, the process is declared as "out-of-control", and the corrective actions must be considered.

The transient probabilities are obtained as follows:

$$Pr(Z_i \in \Omega_1) = \Phi(W) - \Phi(-W) = 2\Phi(W) - 1, \quad (12)$$

$$Pr(Z_i \in \Omega_2) = \Phi(K) - \Phi(W) + \Phi(-W) - \Phi(-K) = 2(\Phi(K) - \Phi(W)), \quad (13)$$

$$Pr(Z_i \in \Omega_3) = \Phi(K) - \Phi(-K) = 2\Phi(K) - 1, \quad (14)$$

To obtain the warning limits coefficient, we use the balanced equation $t_0 = t_2 P_1 + t_1 P_2$, where t_0 is the average sampling interval, P_1 and P_2 are the conditional probabilities of Z_i falling in Ω_1 and Ω_2, respectively, provided that Z_i is "in-control" region (Ω_3). Hence, we have:

$$t_0 = t_2 \frac{Pr(Z_i \in \Omega_1)}{Pr(Z_i \in \Omega_3)} + t_1 \frac{Pr(Z_i \in \Omega_2)}{Pr(Z_i \in \Omega_3)} . \quad (15)$$

By replacing the (12)-(14) in (15) and solving for *W*, the following equation is obtained:

$$W = \Phi^{-1}\left(\frac{2\Phi(L)(t_0 - t_2) + t_1 - t_0}{2(t_1 - t_2)} \right), \quad (16)$$

where $\Phi^{-1}(.)$ denotes the inverse c.d.f. of the $N(0,1)$ distribution.

4. The effect of measurement errors on the VSI EWMA control chart

In this section, the performance of VSI EWMA control chart with a linearly covariate error model over a range of shift sizes is evaluated. By using the W and considering the other parameters $(t_1, t_2, t_0, m, A, B, L)$, we assess the performance of the VSI EWMA control chart in terms of the *ARL* and *ATS*. We use a Markov chain approach to compute the *ARL* and *ATS* for EWMA control chart, first introduced by Lucas and Saccucci [43]. The distance between the control limits is divided into $2p+1$ states. Each state has a width of $2d$, where $2d=\frac{UCL-LCL}{2p+1}$. The midpoint of each state is $f_t,\ t\in\{-p,...,p\}$. The statistic Z_i is in the state t, if $f_t-d\le Z_i\le f_t+d$. When Z_i is within the control limits it is in a transient state, and when Z_i is outside the control limits, it enters the absorbing state. Let **Q** be the $(2p+1,2p+1)$ matrix of transition probabilities in which Q_{gh} is the probability of the movement from state g to state h:

$$Q_{gh}=Pr(f_h-d<Z_i< f_h+d\,|Z_{i-1}=f_g), \quad (17)$$

$$Q_{gh}=Pr(Z_i<f_h+d\,|Z_{i-1}=f_g)- Pr(Z_i<f_h-d\,|Z_{i-1}=f_g). \quad (18)$$

Replacing Z_i with U_i based on (7),

$$Q_{gh}=Pr(U_i<\frac{f_h+d-(1-\lambda)f_j}{\lambda})- Pr(U_i<\frac{f_h-d-(1-\lambda)f_g}{\lambda}). \quad (19)$$

Substituting U_i with $\bar{X}_i$ based on (6), Q_{gh} is obtained as follows:

$$Q_{gh}=Pr(\bar{X}_i<\left(\frac{f_h+d-(1-\lambda)f_g}{\lambda}\right)\sigma_{\bar{X}_i}+\mu_{0_{\bar{X}_i}})- Pr(\bar{X}_i<\left(\frac{f_h-d-(1-\lambda)f_g}{\lambda}\right)\sigma_{\bar{X}_i}+\mu_{0_{\bar{X}_i}}). \quad (20)$$

For computing this probability, the (20) should be standardized, hence, by using (4) and (5), we have:

$$Q_{gh}=Pr(\frac{\bar{X}_i-\mu_{\bar{X}_i}}{\sigma_{\bar{X}_i}}<\frac{\left[\left(\frac{f_h+d-(1-\lambda)f_g}{\lambda}\right)\sigma_{\bar{X}_i}+\mu_{0_{\bar{X}_i}}-\mu_{\bar{X}_i}\right]}{\sigma_{\bar{X}_i}})- Pr(\frac{\bar{X}_i-\mu_{\bar{X}_i}}{\sigma_{\bar{X}_i}}<\frac{\left[\left(\frac{f_h-d-(1-\lambda)f_g}{\lambda}\right)\sigma_{\bar{X}_i}+\mu_{0_{\bar{X}_i}}-\mu_{\bar{X}_i}\right]}{\sigma_{\bar{X}_i}}). \quad (21)$$

Then, we have:

$$Q_{gh} = \Phi\left(\left(\frac{f_h + d - (1-\lambda) f_g}{\lambda}\right) - \frac{B\delta\sigma_0}{\sigma_{\bar{X}_i}}\right) - \Phi\left(\left(\frac{f_h - d - (1-\lambda) f_g}{\lambda}\right) - \frac{B\delta\sigma_0}{\sigma_{\bar{X}_i}}\right). \quad (22)$$

Let $\mathbf{q}$ and $\mathbf{t}$ be the $(2p+1,1)$ vector of initial probabilities and the $(2p+1,1)$ vector of sample sizes, respectively, associated to each transient states, i.e.

$$\mathbf{q_t} = \begin{cases} 0 & if \;\; Z_i \notin (f_t - d, f_t + d] \\ 1 & if \;\; Z_i \in (f_t - d, f_t + d] \end{cases}, \quad (23)$$

$$\mathbf{t_t} = \begin{cases} t_1 & if \;\; LWL \le f_t \le UWL \\ t_2 & otherwise \end{cases}. \quad (24)$$

Finally, the *ARL* and *ATS* of the VSI EWMA control chart can be obtained by using:

$$ARL_1 = \mathbf{q}^T (\mathbf{I} - \mathbf{Q})^{-1} \mathbf{1}, \quad (25)$$

$$ATS_1 = \mathbf{q}^T (\mathbf{I} - \mathbf{Q})^{-1} \mathbf{t}, \quad (26)$$

where $\mathbf{I}$ is the $(2p+1, 2p+1)$ identity matrix and $\mathbf{1}$ is a $(2p+1,1)$ unit column vector.

To evaluate the effect of measurement errors on the performance of the VSI EWMA chart, we used $\lambda = 0.2$ and *L*=2.962 to achieve an in-control *ARL* of 500. Also, we used $2p+1$ states where $p = 105$, based on Maravelakis [33], for the implementation of the Markov chain approach. Also, in our study, we considered both constant measurement errors variance.

5. The effect of measurement errors on the performance of the VSI EWMA control chart

To evaluate the effect of measurement errors on the performance of the VSI EWMA chart, the control limits are divided to 211 transit states. To show the effect of measurement errors on the performance of the VSI EWMA control chart, the *ATS* and *ARL* criteria are used in this paper. We have evaluated the effect of measurement errors on the performance of the VSI EWMA chart with the assumptions $t_1 = \{0.01, 0.1, 0.25, 0.5\}$, $t_2 = \{1.2, 1.5, 1.75, 2\}$, $m = 1$, $\lambda = 0.2$, $k = 2.962$ and $B = 1$. In this situation, the in-control *ATS* and *ARL* are equal to 500. The effect of measurement errors ($\sigma_M^2 = 0, 0.3, 0.7$ and 1) on the performance of the VSI EWMA chart is evaluated and the results are reported in Table 1. The results show that as the measurement errors increase, the *ATS* and *ARL* criteria increase as well. This shows the negative effect of measurement errors on the performance of the proposed control chart.

“Insert Table 1 about here”

Considering $\sigma_M^2 = 1$ and all of the mentioned assumptions, the effect of different values of B $(B = 1, 2, 3, 4)$ on the performance of the VSI EWMA chart is evaluated and reported in Table 2. From Table 2, we can see easily the positive effect of increasing B on the performance of the VSI EWMA chart with measurement errors. Also, the sensitivity analyses on the different number of measurements $(m = 1, 2, 3, 4, 5)$ are shown in Table 3. When m increases, the effect of measurement errors decreases. We can see from Table 3, the multiple measurements decrease the negative effect of measurement errors. In addition, this point is shown with the comparison between the Tables 1 and 4.

"Insert Table 2 about here"

"Insert Table 3 about here"

"Insert Table 4 about here"

6. Comparison between the performance of the VSI and FSI EWMA control chart with measurement errors

In this section to reveal the efficiency of the proposed control chart, the performance of the VSI EWMA control chart is compared with the performance of the FSI EWMA control charts in the presence of measurement errors in terms of ATS_1. Hence, the same situation is provided for control charts, including $\lambda = 0.2, L = 2.962, n = 5, \sigma_M^2 = 1, B = 1, t_1 = 0.5, t_2 = 1.5, t_0 = 1, m = 1$. Note that, for FSI EWMA, chart the sampling interval is equal to $t_1 = t_2 = t_0 = 1$.

The comparison between the performance of the VSI EWMA with FSI EWMA is shown in Figure 1. This figure shows that the VSI EWMA outperforms the FSI EWMA control chart in the presence of measurement errors in detecting mean shifts. For example when $\delta = 0.5$, ATS_1 for VSI EWMA and FSI EWMA is equal to 11.93 and 16.35, respectively.

"Insert Figure 1 about here"

7. An illustrative example

In this section the application of the VSI EWMA control is shown in the example of filling the yogurts cups based on Costa and Castagliola [23]. In this case, the weight of filled cups is the quality characteristic. We generated the new data based on the parameters, $\mu_0 = 124.9$, $\sigma_0^2 = 0.578$ and $\sigma_M^2 = 0.058$ and considering the $\lambda = 0.2$, $L = 2.962$, $t_1 = 0.1$, $t_2 = 1.9$, $n = 5$, $B = 1$, , $m = 1$. Then, based on (5) and (6), we computed the statistics U_i and Z_i, respectively. The upper and lower control limits (± 0.987) and warning limits (± 0.22) are computed based on (7)-(10), respectively. Table 5 and Figure 3 are provided to report the results. We can see easily that the VSI EWMA control chart detects an out-of-control situation in sample 14, hence, the process is declared 'out-of-control' and needs corrective actions.

"Insert Table 5 about here"

"Insert Figure 2 about here"

8. Conclusion and suggestions for future research

In this paper, we investigated the effect of measurement errors on the performance of the VSI EWMA control chart. We modeled the measurement errors using covariate error model. The *ARL* and *ATS* are computed by using a Markov chain approach to evaluate the performance of the VSI EWMA chart with measurement errors. From the reported results, the effect of the measurement errors on the performance of the VSI EWMA control chart is negative. In addition, the sensitivity analyses showed that increasing multiple measurements as well as increasing the parameter *B* reduces the negative effect of the measurement errors on the performance of the VSI EWMA control chart. Also, we compared the performance of the proposed VSI EWMA control chart with FSI EWMA control charts in presence of measurement errors in terms of *ATS* criterion, and the results showed that the VSI EWMA chart outperforms the counterparts in detecting shifts in the process mean.

The effect of measurement errors on the performance of the VSS, VSI, VSSI EWMA control charts with unknown parameters are suggested as future research.

References

[1] Prabhu, S. S., Runger, G. C., & Keats, J. B. (1993). $\bar{X}$ chart with adaptive sample sizes. *The International Journal of Production Research*, 31(12), 2895-2909.

[2] Costa, A. F. (1994). $\bar{X}$ charts with variable sample size. *Journal of Quality Technology*, 26(3), 155-163.

[3] Reynolds, M. R., Amin, R. W., Arnold, J. C., & Nachlas, J. A. (1988). Charts with variable sampling intervals. *Technometrics*, 30(2), 181-192.

[4] Prabhu, S. S., Montgomery, D. C., & Runger, G. C. (1994). A combined adaptive sample size and sampling interval X control scheme. *Journal of Quality Technology*, 26(3), 164-176.

[5] Costa, A. F. (1997). X chart with variable sample size and sampling intervals. *Journal of Quality Technology*, *29*(2), 197-204.

[6] Costa, A. F. (1999). X charts with variable parameters. *Journal of Quality Technology*, 31(4), 408-416.

[7]Tagaras, G. (1998). A survey of recent developments in the design of adaptive control charts. *Journal of quality technology*, *30*(3), 212-231.

[8] Zimmer, L. S., Montgomery, D. C., & Runger, G. C. (2000). Guidelines for the application of adaptive control charting schemes. *International Journal of Production Research*, *38*(9), 1977-1992.

[9] Saccucci, M. S., Amin, R. W., & Lucas, J. M. (1992). Exponentially weighted moving average control schemes with variable sampling intervals. *Communications in Statistics-Simulation and Computation*, 21(3), 627-657.

[10] Reynolds Jr, M. R., & Arnold, J. C. (2001). EWMA control charts with variable sample sizes and variable sampling intervals. *IIE Transactions*, *33*(6), 511-530.

[11] Castagliola, P., Celano, G., & Fichera, S. (2006). Evaluation of the statistical performance of a variable sampling interval R EWMA control chart. *Quality Technology & Quantitative Management*, 3(3), 307-323.

[12] Castagliola, P., Celano, G., Fichera, S., & Giuffrida, F. (2006). A variable sampling interval S2-EWMA control chart for monitoring the process variance. *International Journal of Technology Management*, 37(1-2), 125-146.
[13] Kazemzadeh, R. B., Karbasian, M., & Babakhani, M. A. (2013). An EWMA t chart with variable sampling intervals for monitoring the process mean. *The International Journal of Advanced Manufacturing Technology*, 66(1-4), 125-139.
[14] Tran K.P, Castagliola P., Nguyen TH. and Cuzol A. (2018). "The efficiency of the VSI exponentially weighted moving average median control chart", *Proceeding of 24th ISSAT International Conference on Reliability and Quality in Design*, Toronto, Canada.
[15] Amiri, A., Nedaie, A., & Alikhani, M. (2014). A new adaptive variable sample size approach in EWMA control chart. *Communications in Statistics-Simulation and Computation*, 43(4), 804-812.
[16] Ugaz, W., Sánchez, I., & Alonso, A. M. (2017). Adaptive EWMA control charts with time-varying smoothing parameter. *The International Journal of Advanced Manufacturing Technology*, *93*(9-12), 3847-3858.
[17] Capizzi, G., & Masarotto, G. (2003). An adaptive exponentially weighted moving average control chart. *Technometrics*, *45*(3), 199-207.
[18] Ong, L. V., Teoh, W. L., Khoo, M. B. C., Chong, Z. L., & Yeong, W. C. (2018, November). A study on the variable sampling interval EWMA chart when the process parameters are unknown. *In Journal of Physics: Conference Series* (Vol. 1132, No. 1, p. 012084). IOP Publishing.
[19] Bennett, C. A. (1954). Effect of measurement error on chemical process control. Industrial Quality Control, 10(4), 17-20.
[20] Linna, K. W., & Woodall, W. H. (2001). Effect of measurement error on Shewhart control charts. *Journal of Quality Technology*, 33(2), 213-222.
[21] Linna, K. W., Woodall, W. H., & Busby, K. L. (2001). The performance of multivariate control charts in the presence of measurement error. *Journal of Quality Technology*, *33*(3), 349-355.
[22] Kanazuka, T. (1986). The effect of measurement error on the power of $\bar{X}$ -R charts. *Journal of Quality Technology*, *18*(2), 91-95.
[23] Costa, A. F., & Castagliola, P. (2011). Effect of measurement error and autocorrelation on the X chart. *Journal of Applied Statistics*, *38*(4), 661-673.
[24] Hu, X., Castagliola, P., Sun, J., & Khoo, M. B. (2015). The effect of measurement errors on the synthetic chart. *Quality and Reliability Engineering International*, *31*(8), 1769-1778.
[25] Hu, X., Castagliola, P., Sun, J., & Khoo, M. B. (2016). The performance of variable sample size chart with measurement errors. *Quality and Reliability Engineering International*, *32*(3), 969-983.
[26] Hu, X., Castagliola, P., Sun, J., & Khoo, M. B. C. (2016). Effect of measurement errors on the VSI $\bar{X}$ chart. *European Journal of Industrial Engineering*, *10*(2), 224-242.
[27] Sabahno, H., & Amiri, A. (2017). The effect of measurement errors on the performance of variable sample size and sampling interval control chart. *International Journal of Engineering*, *Transactions A: Basics*, *30*(7), 995-1004.
[28] Sabahno, H., Amiri, A., & Castagliola, P. (2019). Performance of the variable parameters $\bar{X}$ control chart in presence of measurement errors. *Journal of Testing and Evaluation*, 47(1), 480-497.
[29] Sabahno, H., Amiri, A., & Castagliola, P. (2019). Optimal performance of the variable sample sizes Hotelling's T^2 control chart in the presence of measurement errors. *Quality Technology & Quantitative Management*, *16*(5), 588-612.
[30] Sabahno, H., Amiri, A., & Castagliola, P. (2018). Evaluating the effect of measurement errors on the performance of the variable sampling intervals Hotelling's T2 control charts. *Quality and Reliability Engineering International*, *34*(8), 1785-1799.
[31] Maleki, M. R., Amiri, A., & Ghashghaei, R. (2016). Simultaneous monitoring of multivariate process mean and variability in the presence of measurement error with linearly increasing variance under additive covariate model. *International Journal of Engineering-Transactions A: Basics*, *29*(4), 471-480.
[32] Amiri, A., Ghashghaei, R., & Maleki, M. R. (2018). On the effect of measurement errors in simultaneous monitoring of mean vector and covariance matrix of multivariate processes. *Transactions of the Institute of Measurement and Control*, *40*(1), 318-330.
[33] Ghashghaei, R., Bashiri, M., Amiri, A., & Maleki, M. R. (2016). Effect of measurement error on joint monitoring of process mean and variability under ranked set sampling. *Quality and Reliability Engineering International*, *32*(8), 3035-3050.
[34] Maravelakis, P., Panaretos, J., & Psarakis, S. (2004). EWMA chart and measurement error. *Journal of Applied Statistics*, *31*(4), 445-455.
[35] Stemann, D., & Weihs, C. (2001). The EWMA-X-S-control chart and its performance in the case of precise and imprecise data. *Statistical Papers*, *42*(2), 207-223.
[36] Haq, A., Brown, J., Moltchanova, E., & Al-Omari, A. I. (2015). Effect of measurement error on exponentially weighted moving average control charts under ranked set sampling schemes. *Journal of Statistical Computation and Simulation*, *85*(6), 1224-1246.
[37] Yang, S. F., Ho, H. W., & Rahim, M. A. (2007). Effects of measurement error on controlling two dependent process steps. *Economic Quality Control*, *22*(1), 127-139.
[38] Abbasi, S. A. (2010). On the performance of EWMA chart in the presence of two-component measurement error. *Quality Engineering*, *22*(3), 199-213.
[39]Abbasi, S. A. (2016). Exponentially weighted moving average chart and two- component measurement error. *Quality and Reliability Engineering International*, *32*(2), 499-504.
[40] Tang, A., Castagliola, P., Sun, J., & Hu, X. (2018). The effect of measurement errors on the adaptive EWMA chart. *Quality and Reliability Engineering International*, *34*(4), 609-630.
[41] Salmasnia, A., Maleki, M. R., & Niaki, S. T. A. (2018). Remedial measures to lessen the effect of imprecise measurement with linearly increasing variance on the performance of the MAX-EWMAMS scheme. *Arabian Journal for Science and*

Engineering, *43*(6), 3151-3162.
[42] Maleki, M. R., Amiri, A., & Castagliola, P. (2017). Measurement errors in statistical process monitoring: a literature review. *Computers & Industrial Engineering*, *103*, 316-329.
[43] Lucas JM and Saccucci MS. (1990). "Exponentially weighted moving average control schemes: properties and enhancements", *Technimetrics*, 32, 1–12.

Table 1. (ARL_1,$ANOS_1$) when ARL_0=500, k=2.962, λ =0.2, B=1, m=1

t_1,t_2,n	t_1=0.1,t_2=1.5,n=6	t_1=0.01,t_2=2,n=6	t_1=0.25,t_2=1.75,n=3	t_1=0.5,t_2=1.2,n=3
W	W= 0.917	W= 0.668	W= 0.672	W=0.62
$ANOS_0$	1753.6	3751.7	2501.8	3254.2
		$\sigma_M^2 = 0$		
$\delta = 0.1$	(154.8, 72.95)	(154.8, 62.51)	(242.1, 138.7)	(242.1, 175.6)
$\delta = 0.5$	(7.48, 5.16)	(7.48, 4.58)	(13.72, 8.48)	(13.72, 10.87)
$\delta = 1$	(2.94, 2.37)	(2.94, 2.24)	(4.5, 3.46)	(4.5, 3.92)
$\delta = 2$	(1.51, 1.67)	(1.51, 1.73)	(2.09, 1.88)	(2.09, 2.02)
		$\sigma_M^2 = 0.3$		
$\delta = 0.1$	(186.5, 89.81)	(186.5, 77.85)	(276.3, 160.5)	(276.3, 201.7)
$\delta = 0.5$	(9.3, 6.12)	(9.3, 5.31)	(17.69, 10.35)	(17.69, 13.64)
$\delta = 1$	(3.43, 2.69)	(3.43, 2.52)	(5.4, 4.03)	(5.4, 4.64)
$\delta = 2$	(1.75, 1.85)	(1.75, 1.84)	(2.35, 2)	(2.35, 2.19)
		$\sigma_M^2 = 0.7$		
$\delta = 0.1$	(220.9, 108.7)	(220.9, 95.11)	(309.9, 182.4)	(309.9, 227.7)
$\delta = 0.5$	(11.8, 7.32)	(11.8, 6.2)	(23.19, 12.9)	(23.19, 17.4)
$\delta = 1$	(4.05, 3.09)	(4.05, 2.88)	(6.58, 4.75)	(6.58, 5.59)
$\delta = 2$	(1.95, 1.9)	(1.95, 1.8)	(2.69, 2.23)	(2.69, 2.45)
		$\sigma_M^2 = 1$		
$\delta = 0.1$	(242.1, 120.6)	(242.1, 106.08)	(329.1, 195.1)	(329.1, 242.7)
$\delta = 0.5$	(13.72, 8.19)	(13.72, 7.26)	(27.42, 14.87)	(27.42, 20.26)
$\delta = 1$	(4.50, 3.39)	(4.50, 3.14)	(7.4, 5.2)	(7.4, 6.2)
$\delta = 2$	(2.09, 1.93)	(2.09, 1.81)	(2.9, 2.4)	(2.9, 2.6)

Table 2. (ARL_1,ATS_1) when ARL_0=ATS_0=500, k=2.962, λ =0.2, B=1, $\sigma_M^2 = 1$

t_1,t_2,n	t_1=0.1,t_2=1.5,n=6	t_1=0.01,t_2=2,n=6	t_1=0.25,t_2=1.75,n=3	t_1=0.5,t_2=1.2,n=3
W	W= 0.917	W= 0.668	W= 0.672	W=0.62
		$m = 1$		
$\delta = 0.1$	(242.1, 120.6)	(242.1, 106.08)	(329.1, 195.1)	(329.1, 242.7)
$\delta = 0.5$	(13.72, 8.19)	(13.72, 7.26)	(27.42, 14.87)	(27.42, 20.26)
$\delta = 1$	(4.50, 3.39)	(4.50, 3.14)	(7.4, 5.2)	(7.4, 6.2)
$\delta = 2$	(2.09, 1.93)	(2.09, 1.81)	(2.9, 2.4)	(2.9, 2.6)
		$m = 2$		
$\delta = 0.1$	(204.7, 99.72)	(204.7, 86.90)	(294.4, 172.3)	(294.4, 215.7)
$\delta = 0.5$	(10.54, 6.73)	(10.54, 5.7)	(20.42, 11.62)	(20.42, 15.52)
$\delta = 1$	(3.74, 2.89)	(3.74, 2.7)	(5.99, 4.40)	(5.99, 5.12)
$\delta = 2$	(1.86, 1.89)	(1.86, 1.83)	(2.52, 2.11)	(2.52, 2.32)
		$m = 3$		
$\delta = 0.1$	(189.7, 91.53)	(189.7, 79.42)	(279.5, 162.6)	(279.5, 204.2)
$\delta = 0.5$	(9.50, 6.22)	(9.50, 5.39)	(18.14, 10.56)	(18.14, 13.95)
$\delta = 1$	(3.48, 2.72)	(3.48, 2.5)	(5.5, 4.1)	(5.5, 4.7)
$\delta = 2$	(1.77, 1.86)	(1.77, 1.84)	(2.38, 2.02)	(2.38, 2.2)
		$m = 4$		
$\delta = 0.1$	(181.6, 87.17)	(181.6, 75.44)	(271.2, 157.3)	(271.2, 197.8)
$\delta = 0.5$	(8.99, 5.97)	(8.99, 5.2)	(17.02, 10.04)	(17.02, 13.18)
$\delta = 1$	(3.35, 2.64)	(3.35, 2.48)	(5.25, 3.94)	(5.25, 4.52)
$\delta = 2$	(1.72, 1.83)	(1.72, 1.84)	(2.30, 1.98)	(2.30, 2.1)
		$m = 5$		
$\delta = 0.1$	(176.6, 84.47)	(176.6, 72.98)	(265.9, 153.9)	(265.9, 193.8)
$\delta = 0.5$	(8.69, 5.81)	(8.69, 5.08)	(16.35, 9.72)	(16.35, 12.72)
$\delta = 1$	(3.27, 2.58)	(3.27, 2.43)	(5.10, 3.85)	(5.1, 4.4)
$\delta = 2$	(1.68, 1.81)	(1.68, 1.83)	(2.26, 1.95)	(2.2, 2.1)

Table 3. (ARL_1,ATS_1) when ARL_0=ATS_0=500, k=2.962, λ =0.2, B=1, $\sigma_M^2 = 1$

t_1,t_2,n W	$t1$=0.1,$t2$=1.5,n=6 W= 0.917	t_1=0.01,t_2=2,n=6 W= 0.668	t_1=0.25,t_2=1.75,n=3 W= 0.672	t_1=0.5,t_2=1.2,n=3 W= 1.06
		$B = 1$		
$\delta = 0.1$	(242.1, 120.6)	(242.1, 106.08)	(329.1, 195.1)	(329.1, 242.7)
$\delta = 0.5$	(13.72, 8.19)	(13.72, 7.26)	(27.42, 14.87)	(27.42, 20.26)
$\delta = 1$	(4.50, 3.39)	(4.50, 3.14)	(7.4, 5.2)	(7.4, 6.2)
$\delta = 2$	(2.09, 1.93)	(2.09, 1.81)	(2.9, 2.4)	(2.9, 2.6)
		$B = 2$		
$\delta = 0.1$	(181.6, 87.17)	(181.6, 75.44)	(271.2, 157.3)	(271.2, 197.8)
$\delta = 0.5$	(8.99, 5.97)	(8.99, 5.20)	(17.02, 10.04)	(17.02, 13.18)
$\delta = 1$	(3.35, 2.64)	(3.35, 2.48)	(5.25, 3.94)	(5.25, 4.52)
$\delta = 2$	(1.72, 1.83)	(1.72, 1.84)	(2.30, 1.98)	(2.30, 2.16)
		$B = 3$		
$\delta = 0.1$	(167.3, 79.49)	(167.3, 68.45)	(255.9, 147.4)	(255.9, 186.1)
$\delta = 0.5$	(8.15, 5.53)	(8.15, 4.86)	(15.17, 9.17)	(15.17, 11.89)
$\delta = 1$	(3.12, 2.49)	(3.12, 2.35)	(4.83, 3.68)	(4.83, 4.19)
$\delta = 2$	(1.61, 1.76)	(1.61, 1.80)	(2.18, 1.91)	(2.18, 2.08)
		$B = 4$		
$\delta = 0.1$	(161.9, 76.68)	(161.9, 65.89)	(250.1, 143.7)	(250.1, 181.7)
$\delta = 0.5$	(7.86, 5.37)	(7.86, 4.74)	(14.54, 8.87)	(14.54, 11.45)
$\delta = 1$	(3.04, 2.44)	(3.04, 2.30)	(4.69, 3.58)	(4.69, 4.07)
$\delta = 2$	(1.57, 1.73)	(1.57, 1.78)	(2.14, 1.90)	(2.14, 2.05)

Table 4. (ARL_1,ATS_1) when ARL_0=ATS_0=500, k=2.962, λ =0.2, B=1, m= 5

t_1,t_2,n W	t_1=0.1,t_2=1.5,n=6 W=0.917	t_1=0.01,t_2=2,n=6 W=0.668	t_1=0.25,t_2=1.75,n=3 W=0.672	t_1=0.5,t_2=1.2,n=3 W=1.06
		$\sigma_M^2 = 0$		
$\delta = 0.1$	(154.8, 72.95)	(154.8, 62.51)	(242.1, 138.7)	(242.1, 175.6)
$\delta = 0.5$	(7.48, 5.16)	(7.48, 4.58)	(13.72, 8.48)	(13.72, 10.87)
$\delta = 1$	(2.94, 2.37)	(2.94, 2.24)	(4.50, 3.46)	(4.50, 3.92)
$\delta = 2$	(1.51, 1.67)	(1.51, 1.73)	(2.09, 1.88)	(2.09, 2.02)
		$\sigma_M^2 = 0.3$		
$\delta = 0.1$	(161.7, 76.53)	(161.7, 65.76)	(249.8, 143.5)	(249.8, 181.4)
$\delta = 0.5$	(7.84, 5.36)	(7.84, 4.73)	(14.50, 8.85)	(14.50, 11.42)
$\delta = 1$	(3.04, 2.44)	(3.04, 2.30)	(4.68, 3.58)	(4.68, 4.06)
$\delta = 2$	(1.57, 1.72)	(1.57, 1.77)	(2.14, 1.89)	(2.14, 2.05)
		$\sigma_M^2 = 0.7$		
$\delta = 0.1$	(170.4, 81.14)	(170.4, 69.94)	(259.3, 149.6)	(259.3, 188.7)
$\delta = 0.5$	(8.32, 5.62)	(8.32, 2.33)	(15.56, 9.35)	(15.56, 12.16)
$\delta = 1$	(3.17, 2.52)	(3.17, 2.37)	(4.92, 3.73)	(4.92, 4.26)
$\delta = 2$	(1.64, 1.78)	(1.64, 1.81)	(2.21, 1.93)	(2.21, 2.10)
		$\sigma_M^2 = 1$		
$\delta = 0.1$	(176.6, 84.47)	(176.6, 72.98)	(265.9, 153.9)	(265.9, 193.8)
$\delta = 0.5$	(8.69, 5.81)	(8.69, 4.93)	(16.35, 9.72)	(16.35, 12.72)
$\delta = 1$	(3.27, 2.58)	(3.27, 2.43)	(5.10, 3.85)	(5.10, 4.40)
$\delta = 2$	(1.68, 1.81)	(1.68, 1.83)	(2.26, 1.95)	(2.26, 2.13)

Table 5. The sample interval *t* and statistics $\bar{X}_i$, U_i, Z_i and process status for simulated data generated based on the parameters reported by Costa and Castagliola [23]

Time(*i*)	t_i	$\bar{X}_i$	U_i	Z_i	Status
1	1.9	124.86	-0.09	-0.01	In-control
2	1.9	124.69	-0.58	-0.13	In-control
3	0.1	125.49	1.66	0.22	In-control
4	0.1	125.11	0.59	0.30	In-control
5	0.1	125.58	1.92	0.62	In-control
6	0.1	125.53	1.79	0.35	In-control
7	1.9	125.59	1.94	0.38	In-control
8	0.1	125.69	2.23	0.44	In-control
9	0.1	124.88	-0.03	-0.007	In-control
10	1.9	125.45	1.55	0.31	In-control
11	0.1	125.54	1.81	0.36	In-control
12	0.1	124.79	-0.28	-0.05	In-control
13	0.1	125.82	2.60	0.52	In-control
14	**0.1**	**126.68**	**5.00**	**1.001**	**Out-of-control**

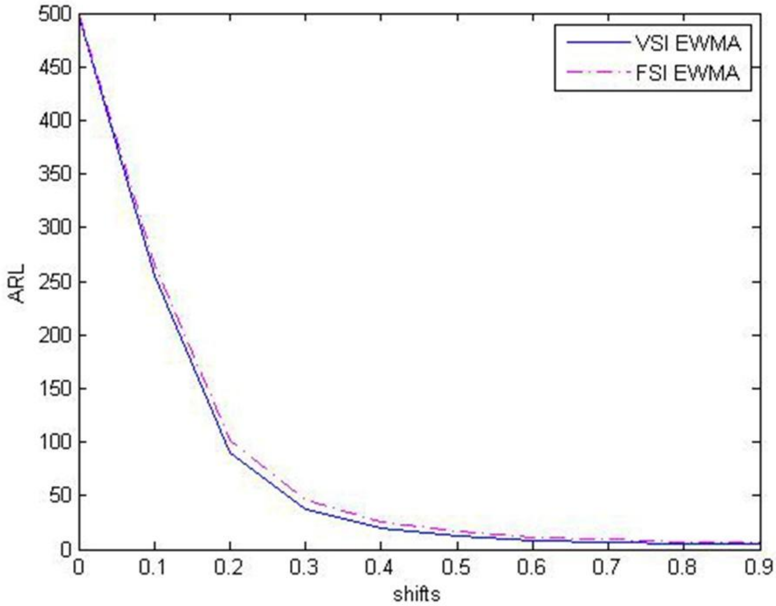

Figure 1. The performance comparison of VSI with FSI EWMA control charts with measurement errors

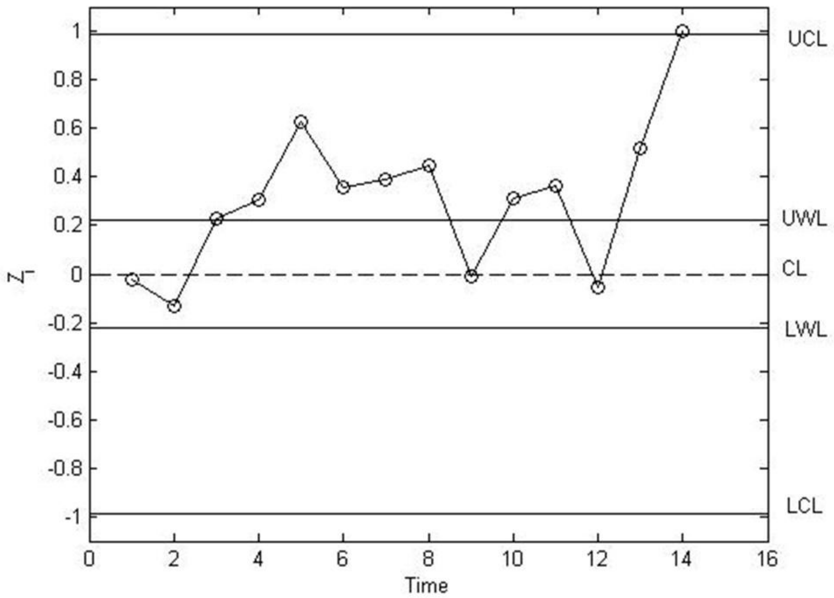

Figure 2. VSI EWMA control chart for the real case example